Geometry Unit 11

12-3: Area and Volume of Cylinders and Cones
12.1 – 12.2 Warm-up

Complete the worksheet provided

*Keep the backside for your notes.
Cylinders and Cones

• **Content Objective**: Students will be able to compare and contrast cylinders and cones to prisms and pyramids to determine their area and volume equations.

• **Language Objective**: Students will be able to use equations to solve for the areas and volume of cylinders and cones.
Cylinders – A Introduction

For the following diagrams, compare and contrast a Right Prism to a Cylinder. Discuss your thoughts in your group, and take notes of your thoughts in the space provided.

For the discussion, focus on these questions:

- What do you notice about each the cylinder?
- How do its parts compare to that of the prism? How do they differ?
Cylinders

- A Cylinder shares similar properties to the right prism.
- It has two bases, and these bases are always circles.
- The line segment joining the bases is the height, $h$.
- The radius of the base is also the radius of the cylinder.

*How would the lateral area and volume of a cylinder be similar to those of Prisms?
Cylinders – Lateral Area

**Theorem 12-5:** The lateral area of a cylinder equals the circumference of a base time the height of the cylinder.

Equation: \( L.A. = 2\pi rh \)

*Total Area: \( T.A. = L.A. + 2B \)
Cylinders – Volume

**Theorem 12-6:** The volume of a cylinder equals the area of a base time the height of the cylinder.

Equation: \( V = \pi r^2 h \)
Cylinders – Examples

For the following Cylinders, find the

a.) Lateral Area

b.) Total Area

c.) Volume

1.)

\[ r = 8 \]

\[ h = 7 \]

\[ B = \pi(8^2) = 64\pi \]
<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Total Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L.A. = 2\pi rh )</td>
<td>( T.A. = L.A. + 2B )</td>
<td>( V = \pi r^2 h )</td>
</tr>
<tr>
<td>( = 2\pi \times 8 \times 7 )</td>
<td>( = 112\pi + 2(64\pi) )</td>
<td>( = 64\pi \times 7 )</td>
</tr>
<tr>
<td>( = 112\pi )</td>
<td>( = 112\pi + 128\pi )</td>
<td>( = 448\pi )</td>
</tr>
</tbody>
</table>
Cylinders – Examples

For the following Cylinders, find the

a.) Lateral Area
b.) Total Area
c.) Volume

2.)  

\[ r = 6 \]
\[ h = 15 \]
\[ B = \pi(6^2) = 36\pi \]
## Cylinder Example #2 Solution

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Total Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L.A. = 2\pi rh )</td>
<td>( T.A. = L.A. + 2B )</td>
<td>( V = \pi r^2 h )</td>
</tr>
<tr>
<td>( = 2\pi \times 6 \times 15 )</td>
<td>( = 180\pi + 2(36\pi) )</td>
<td>( = 36\pi \times 15 )</td>
</tr>
<tr>
<td>( = 180\pi )</td>
<td>( = 180\pi + 72\pi )</td>
<td>( = 540\pi )</td>
</tr>
<tr>
<td></td>
<td>( = 252\pi )</td>
<td></td>
</tr>
</tbody>
</table>
Cones

For the following diagrams, compare and contrast a Regular Pyramid to a Cone. Discuss your thoughts in your group, and take notes of your thoughts in the space provided.

For the discussion, focus on these questions:
• What do you notice about each the cone?
• How do its parts compare to that of the pyramid? How do they differ?
Cones

- A Cone shares similar properties to the regular pyramid.
- It has a single base, and that base will always be a circle.
- The line segment joining the vertex to the base is the height, \( h \).
- The segment joining the vertex to an end of the diameter of the base is the slant height, \( l \).
- The radius of the base is also the radius of the cylinder.

*How would the lateral area and volume of a cone be similar to those of pyramids?
Cones – Lateral Area

**Theorem 12-7:** The lateral area of a cone equals half the circumference of the base time the slant height.

Equation: \( L.A. = \frac{1}{2} \times 2\pi rl \)

Or

\( L.A. = \pi rl \)

*Total Area: \( T.A. = L.A. + B \)
Theorem 12-8: The volume of a cone equals one third the area of the base times the height of the cone.

Equation: $V = \frac{1}{3}\pi r^2 h$
Cones – Examples

For the following Cones, find the

a.) Lateral Area
b.) Total Area
c.) Volume

1.)

$B = \pi (3^2) = 9\pi$

$r = 3$
$l = 14$
$h = 10$
## Cone Example #1 Solution

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Total Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L.A. = \pi rl$</td>
<td>$T.A. = L.A. + B$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
</tr>
<tr>
<td>$= \pi \times 3 \times 14$</td>
<td>$= 42\pi + 9\pi$</td>
<td>$\frac{1}{3} \times 9\pi \times 10$</td>
</tr>
<tr>
<td>$= 42\pi$</td>
<td>$= 51\pi$</td>
<td>$= 30\pi$</td>
</tr>
</tbody>
</table>
Cones – Examples

For the following Cones, find the

a.) Lateral Area
b.) Total Area
c.) Volume

1. \( l = 13 \)
2. \( r = 5 \)
\( h = 12 \)

\[ B = \pi (5^2) = 25\pi \]
**Cone Example #1 Solution**

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Total Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L. A. = \pi rl )</td>
<td>( T. A. = L. A. + B )</td>
<td>( V = \frac{1}{3} \pi r^2 h )</td>
</tr>
<tr>
<td>( = \pi \times 5 \times 13 )</td>
<td>( = 65\pi + 25\pi )</td>
<td>( = \frac{1}{3} \times 25\pi \times 12 )</td>
</tr>
<tr>
<td>( = 65\pi )</td>
<td>( = 90\pi )</td>
<td>( = 100\pi )</td>
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</tbody>
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