Geometry Unit 10

11-4: Areas of Regular Polygons
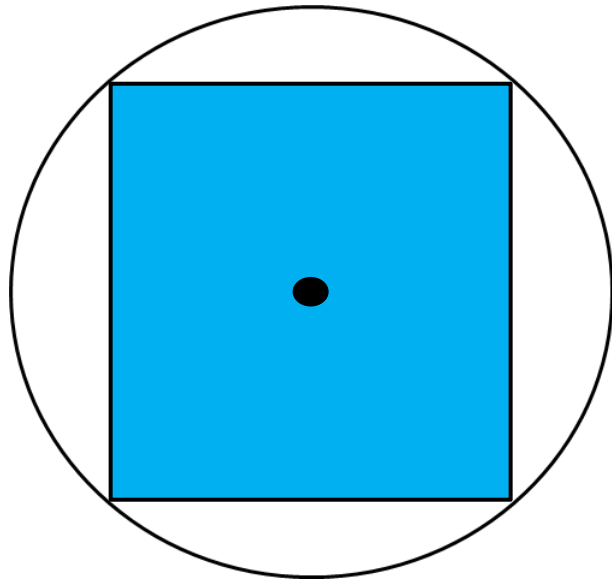
Areas of Regular Polygons

- **<u>Content Objective</u>**: Students will be able to use theorems and equations to solve for the area of regular polygons.

- **<u>Language Objective</u>**: Students will be able to identify various polygons, as well as find their area using equations.
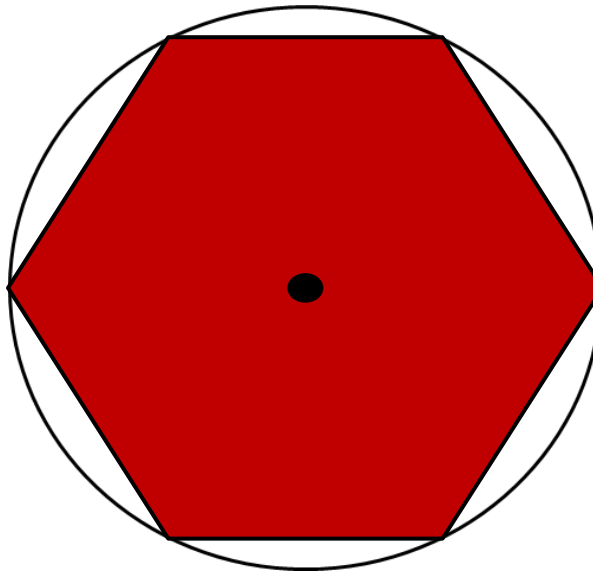
Regular Polygons

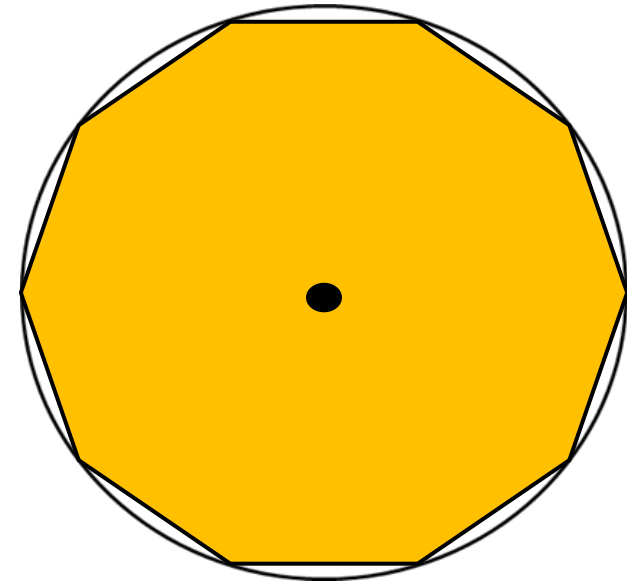- Given any circle, you can inscribe in it a regular polygon of any number of sides.

**Square – 4 Sides**   **Hexagon – 6 Sides**   **Decagon – 10 Sides**

Regular Polygons - Vocabulary

- The relationship between circles and polygons leads to the following terms:

1. The **center** of a regular polygon is the center of the circle.

2. The **radius** of the regular polygon is the radius of the circle.

3. A **central angle** of a regular polygon is an angle formed by two radii drawn at the center.

4. The **apothem** of a regular polygon is the perpendicular distance from the center of the polygon to a side.

Regular Polygons - Vocabulary

- Example of each term (on an Octagon)

Radius

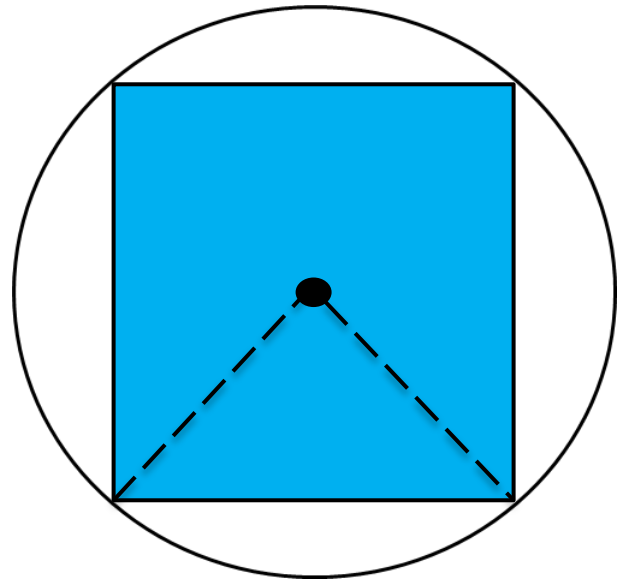Center

Central Angle

Apothem
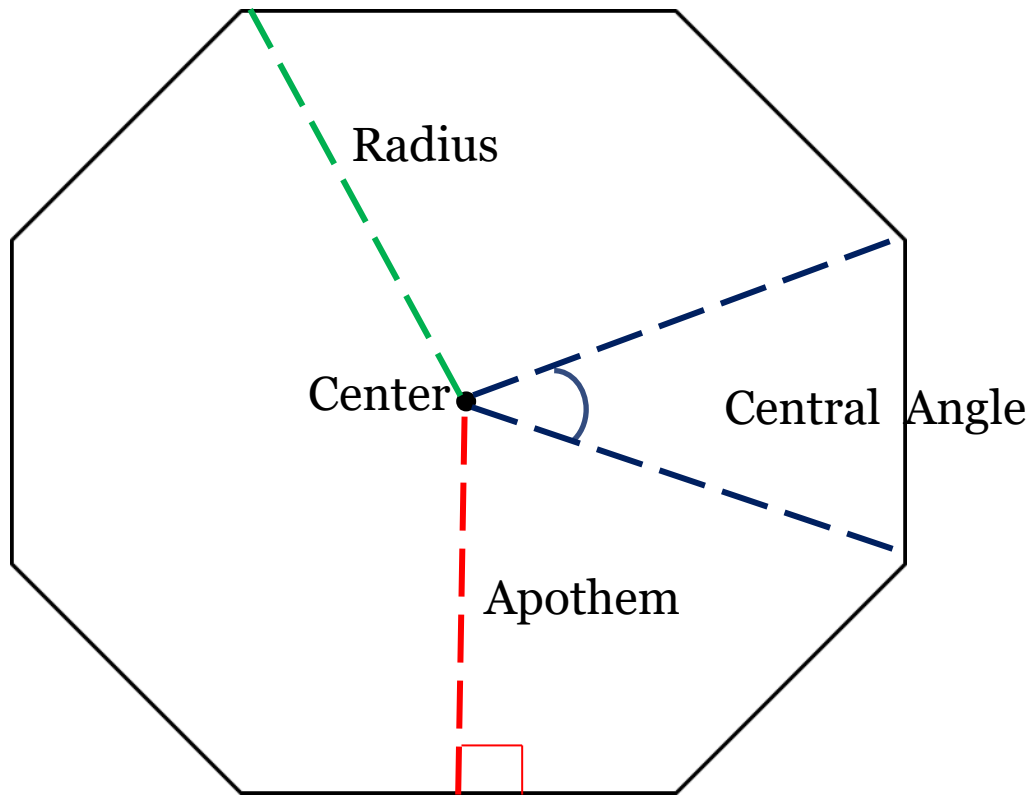
Regular Polygons

- When a central angle is made using two radii that connect to vertices of the polygon, it splits the measure of a circle into as many parts as there are sides.
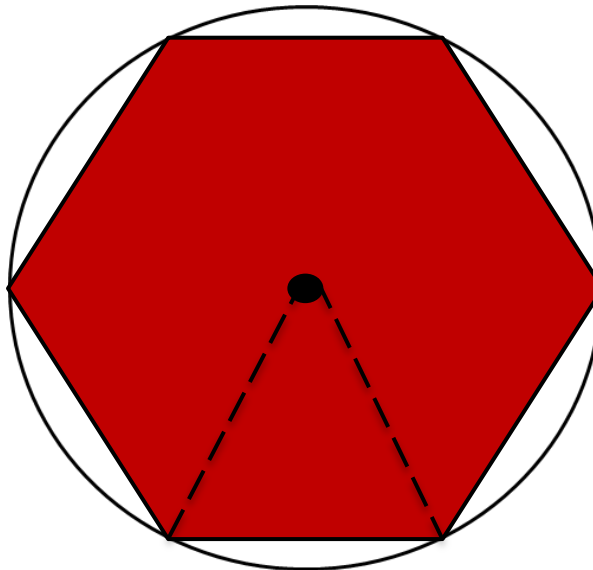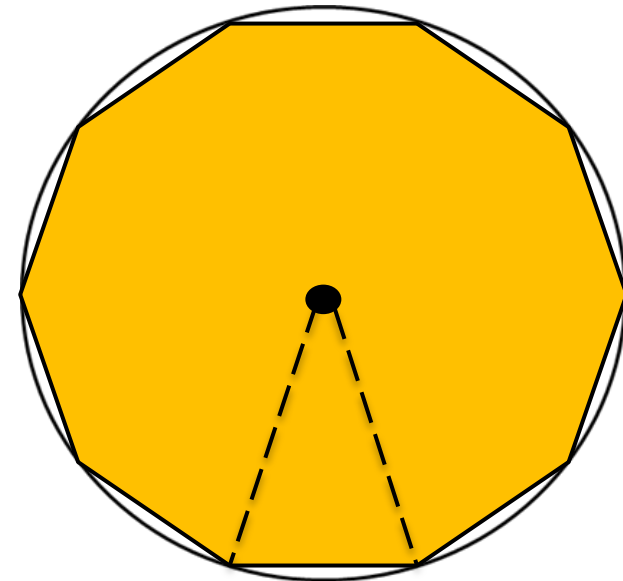
Square
Central Angle: 90°

Hexagon
Central Angle: 60°
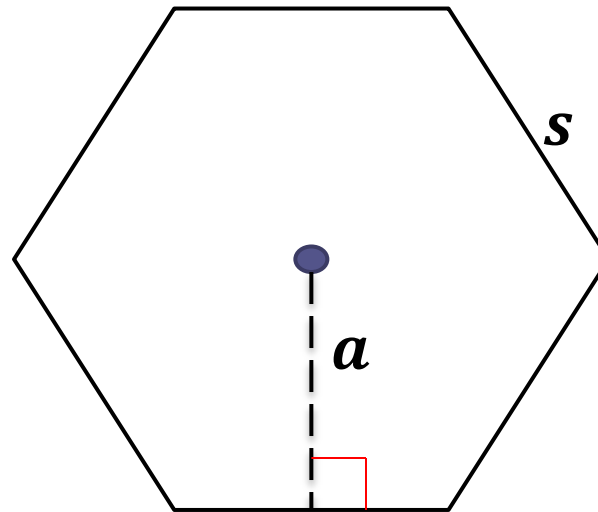
Decagon
Central Angle: 90°

Area of Regular Polygons

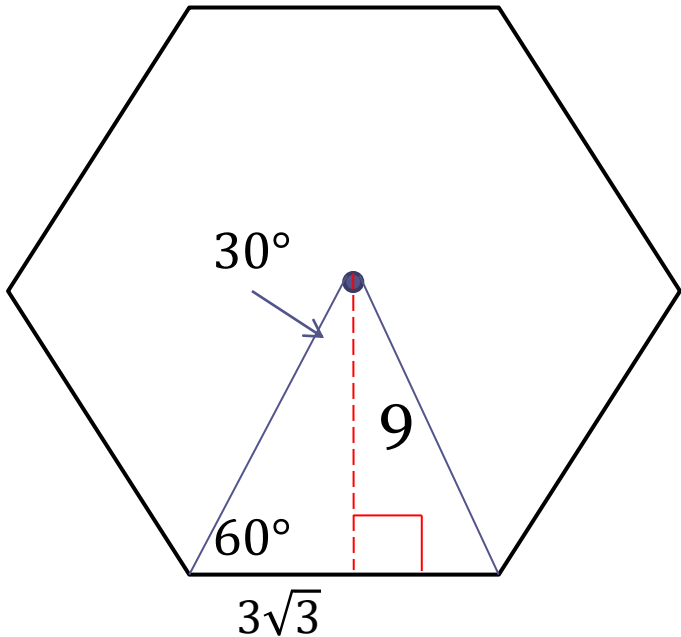**Theorem 11-6:** The area of a regular polygon is equal to half the product of the apothem and the perimeter.

Equation: $A = \frac{1}{2}ap$

$p = s \times \#of\ sides$

Practice

- Find the area of the polygon
  - Regular Hexagon with Apothem 9

To solve, we also need the perimeter.

We can use the apothem to find the measure of one side

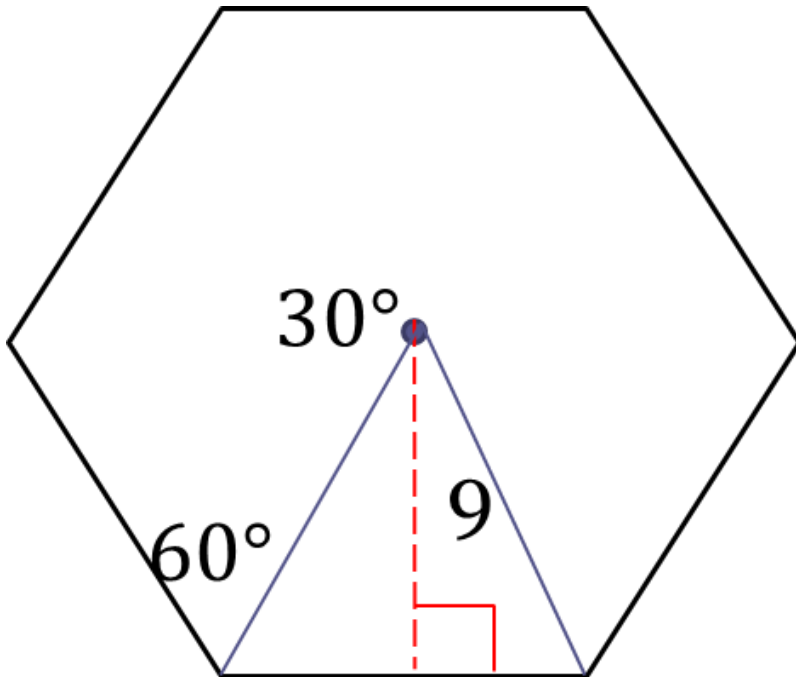We start by constructing a $30° - 60° - 90°$ triangle.

We can use the rules for this type of triangle to find the measure of half the side.

$$\frac{1}{2}s = 3\sqrt{3}, \qquad s = 6\sqrt{3}$$

$$p = 36\sqrt{3}$$

(Hexagon figure with labels: $30°$, 9, $60°$, $3\sqrt{3}$)

Practice

- Find the area of the polygon
  - Regular Hexagon with Apothem 9

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 9 \times 36\sqrt{3}$$

$$\boldsymbol{A = 162\sqrt{3}}$$

30°

9

60°

Practice

- Find the area of the polygon
  - Regular decagon inscribed in a circle with radius 10

We will need to use trig to find the side and apothem.
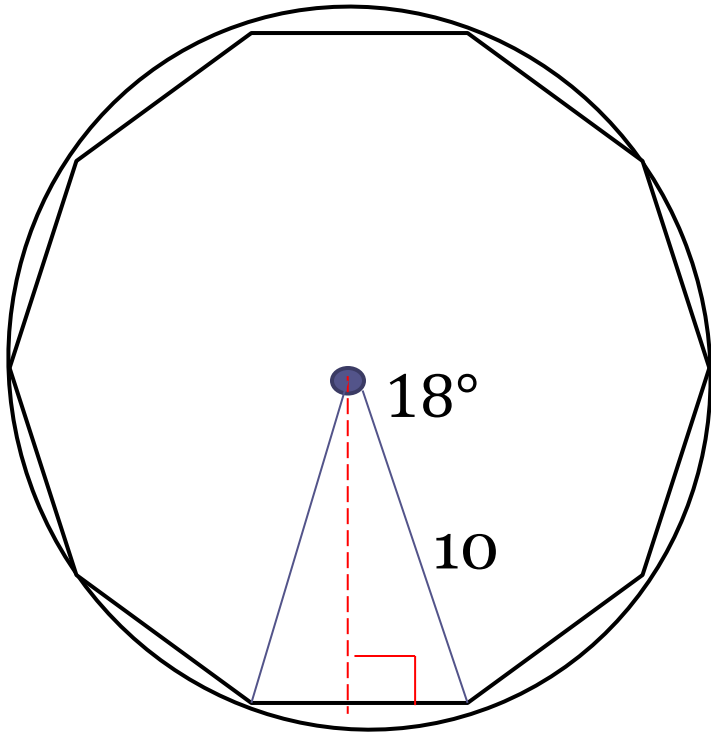
Apothem: $\quad \cos 18 = \dfrac{a}{10}$

$\qquad a = 10 \times \cos 18 = 9.5106$

Side: $\quad \sin 18 = \dfrac{h}{10}$

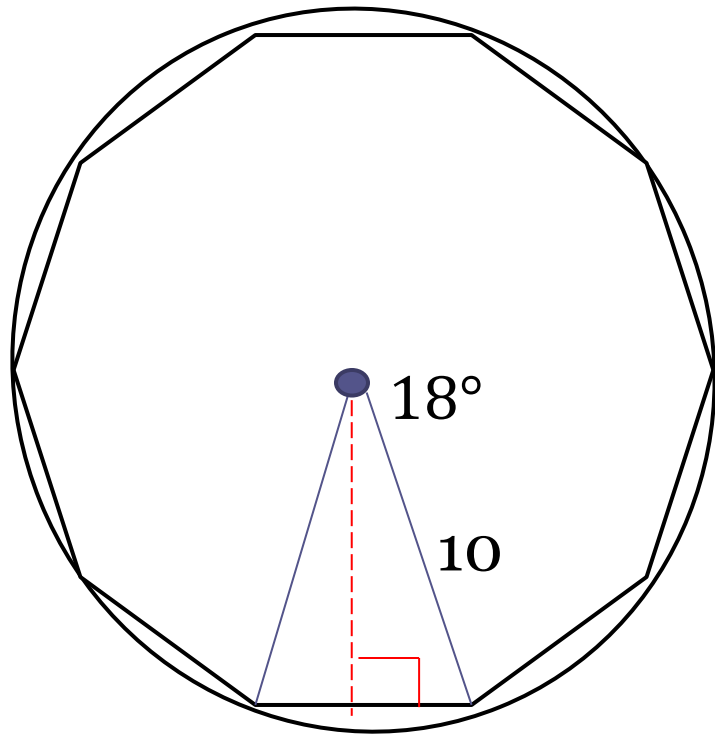$\qquad h = 10 \times \sin 18 = 3.0902$

$\qquad s = h \times 2 = 6.1804$

Finding the area can now be done in one of two ways...

18°

10

Practice

- Find the area of the polygon
  - Regular decagon inscribed in a circle with radius 10

Method 1:

$$A = 10 \times area\ of\ the\ triangle$$

$$A = 10 \times \frac{1}{2}(9.5106)(6.1804)$$

$$\boldsymbol{A \approx 294}$$
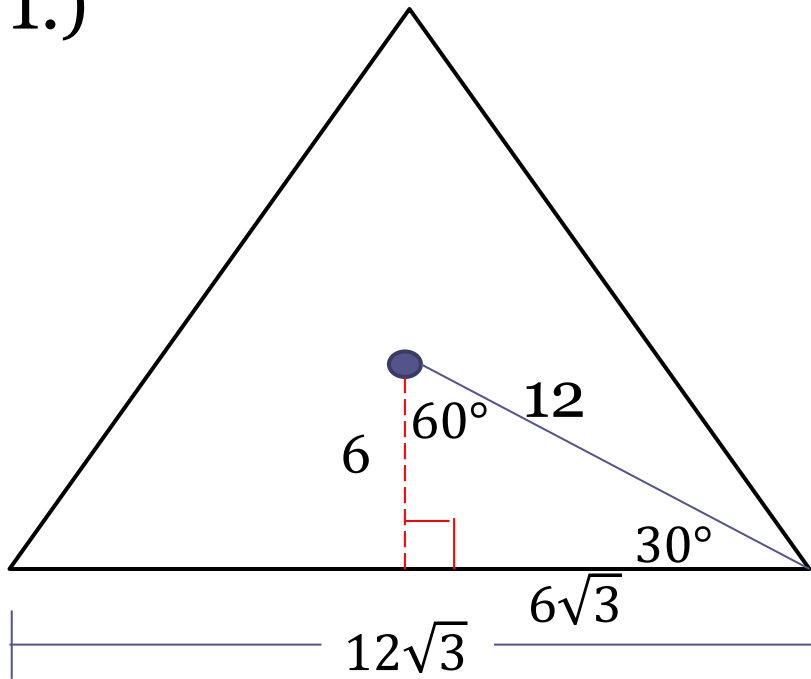
Method 2:

$$A = \frac{1}{2}ap$$

$$A = 1/2(9.5106)(10 \times 6.1804)$$

$$\boldsymbol{A \approx 294}$$

Group Practice

- Find the area of the polygons in your groups

1.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 6 \times 36\sqrt{3}$$

$$\boldsymbol{A = 108\sqrt{3}}$$

Diagram labels: $60°$, 12, 6, $30°$, $6\sqrt{3}$, $12\sqrt{3}$

$$p = 12\sqrt{3} \times 3 = 36\sqrt{3}$$

Group Practice

- Find the area of the polygons in your groups

2.)
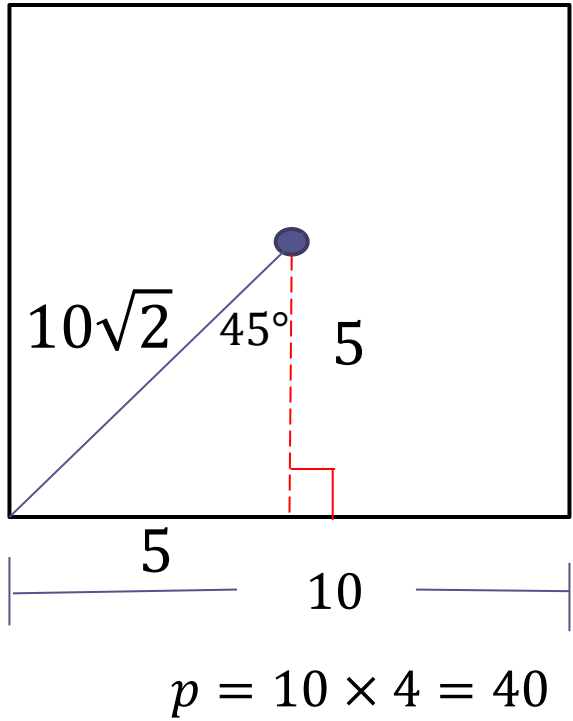
Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 5 \times 40$$

$$\boldsymbol{A = 100}$$

$10\sqrt{2}$ $\quad 45°$ $\quad 5$

5

10

$$p = 10 \times 4 = 40$$

Group Practice

- Find the area of the polygons in your groups

3.)
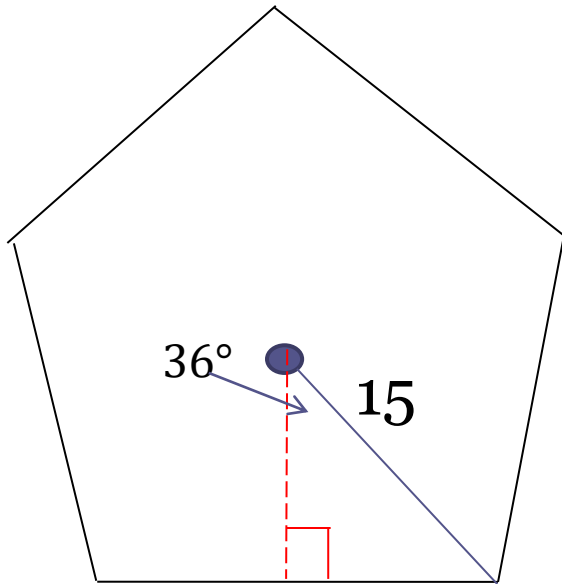
We will need to use trig to find the side and apothem.

Apothem: $\cos 36 = \dfrac{a}{15}$

$a = 15 \times \cos 36 = 12.1353$

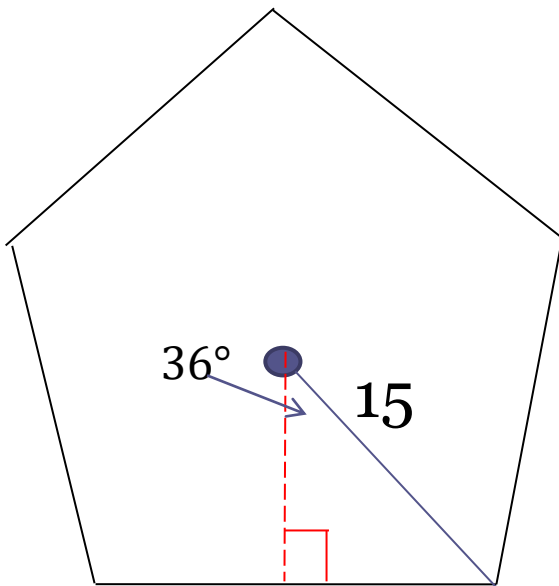Side: $\sin 36 = \dfrac{h}{15}$

$h = 15 \times \sin 36 = 8.8168$

$s = h \times 2 = 17.6336$

Solution on next slide..

Group Practice

- Find the area of the polygons in your groups
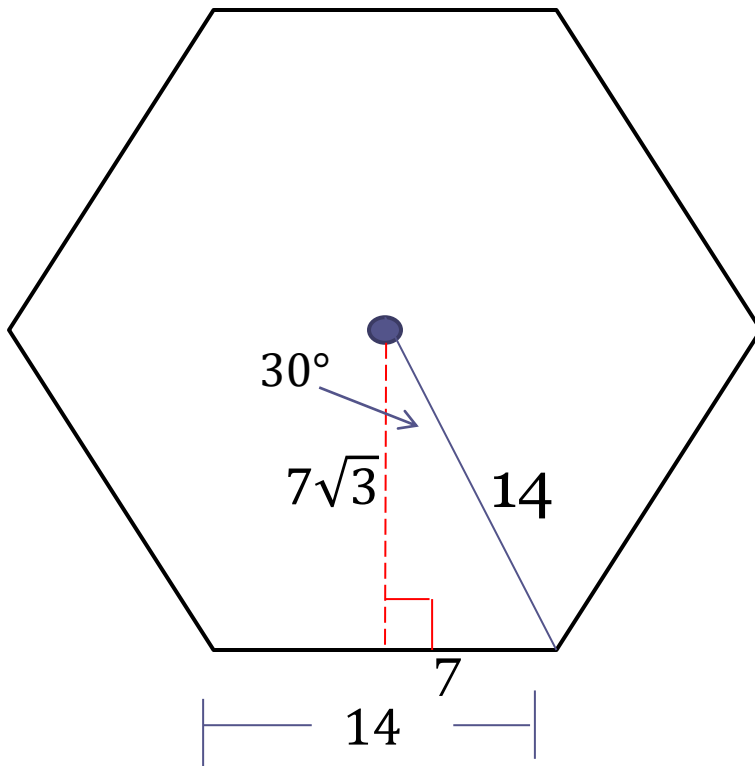
3.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 12.1353 \times (5 \times 17.6336)$$

$$A = 534.9726$$

36°

15

Group Practice

- Find the area of the polygons in your groups
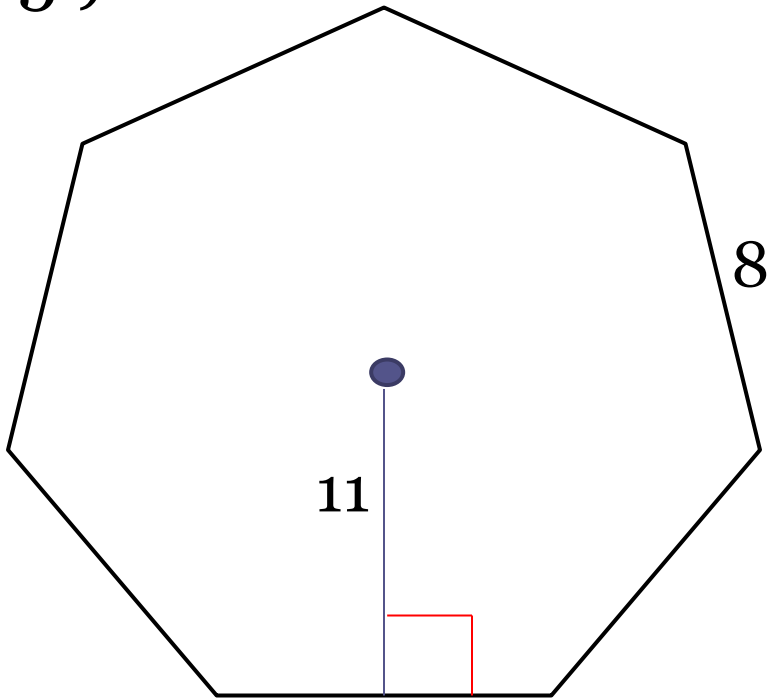
4.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 7\sqrt{3} \times 84$$

$$\boldsymbol{A = 294\sqrt{3}}$$

$$p = 14 \times 6 = 84$$

Group Practice

- Find the area of the polygons in your groups

5.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 11 \times 56$$

$A = 308$

8

11

$$p = 8 \times 7 = 56$$

Group Practice

- Find the area of the polygons in your groups

6.)
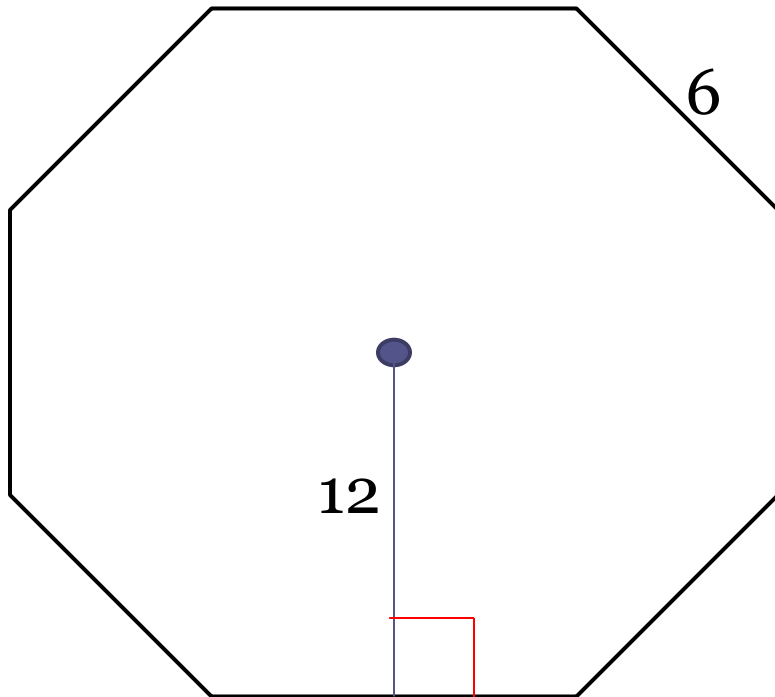
Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 12 \times 48$$

$$\boldsymbol{A = 288}$$

6

12
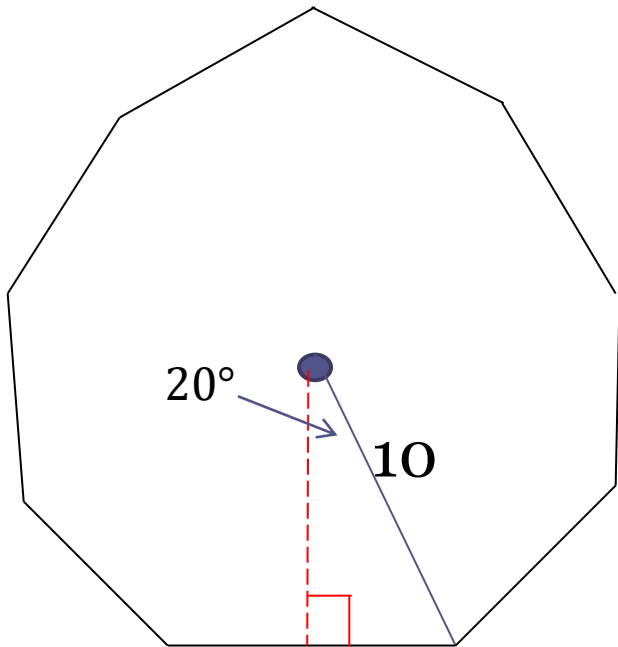
$$p = 6 \times 8 = 48$$

Group Practice

- Find the area of the polygons in your groups

7.)

We will need to use trig to find the side and apothem.

Apothem: $\cos 20 = \dfrac{a}{10}$

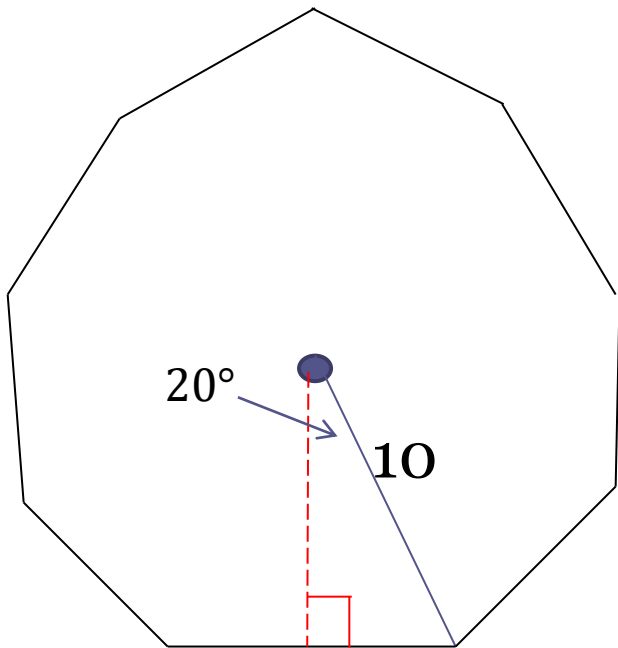$a = 10 \times \cos 20 = 9.3979$

Side: $\sin 20 = \dfrac{h}{10}$

$h = 10 \times \sin 20 = 3.4202$

$s = 6.8404$

Solution on next slide..

Group Practice

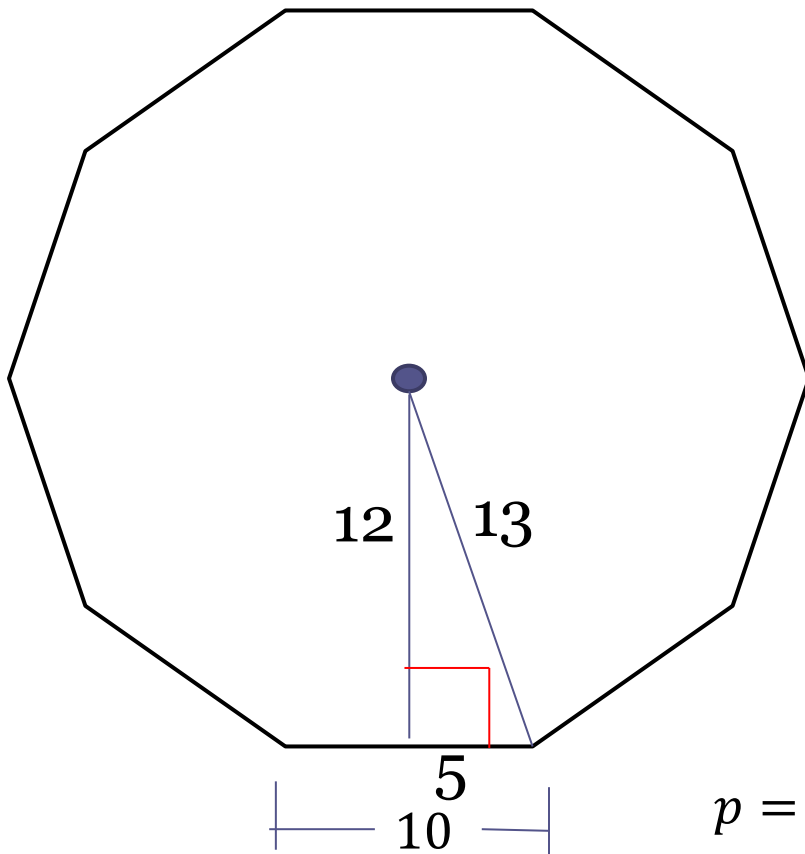- Find the area of the polygons in your groups

7.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 9.3979 \times (9 \times 6.8404)$$

$$\boldsymbol{A = 289.2843}$$

Group Practice

- Find the area of the polygons in your groups

8.)

Solution:

$$A = \frac{1}{2}ap$$

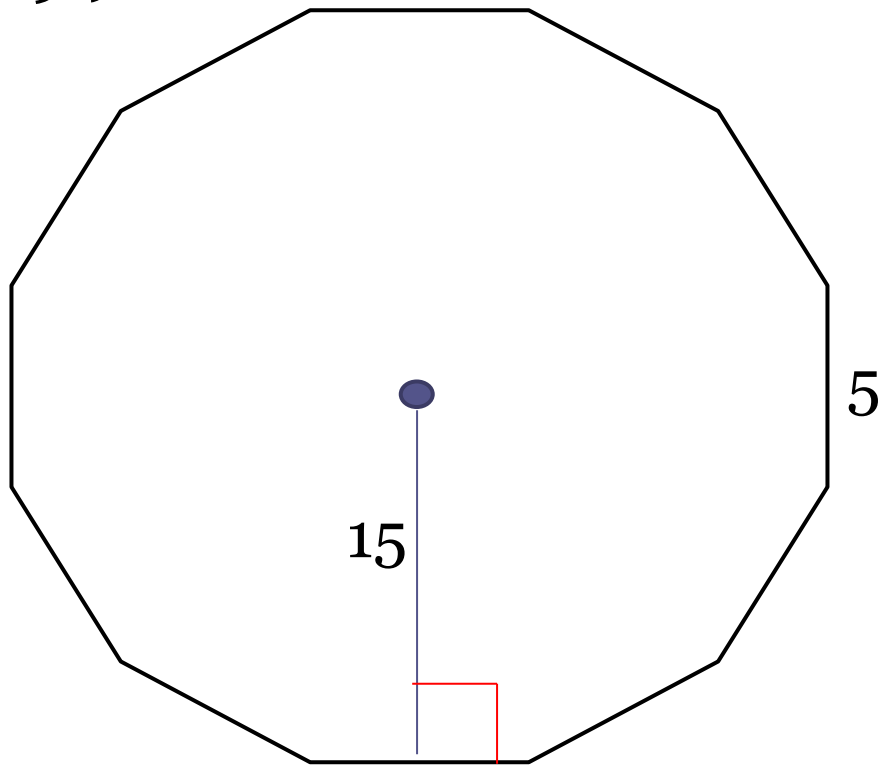$$A = \frac{1}{2} \times 12 \times 100$$

$$\boldsymbol{A = 600}$$

12   13

5

10

$$p = 10 \times 10 = 100$$

Group Practice

- Find the area of the polygons in your groups

9.)

Solution:

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2} \times 15 \times 60$$

$$\boldsymbol{A = 450}$$

$$p = 5 \times 12 = 60$$