GEOMETRY UNIT 11

12-1: Area and Volume of Prisms

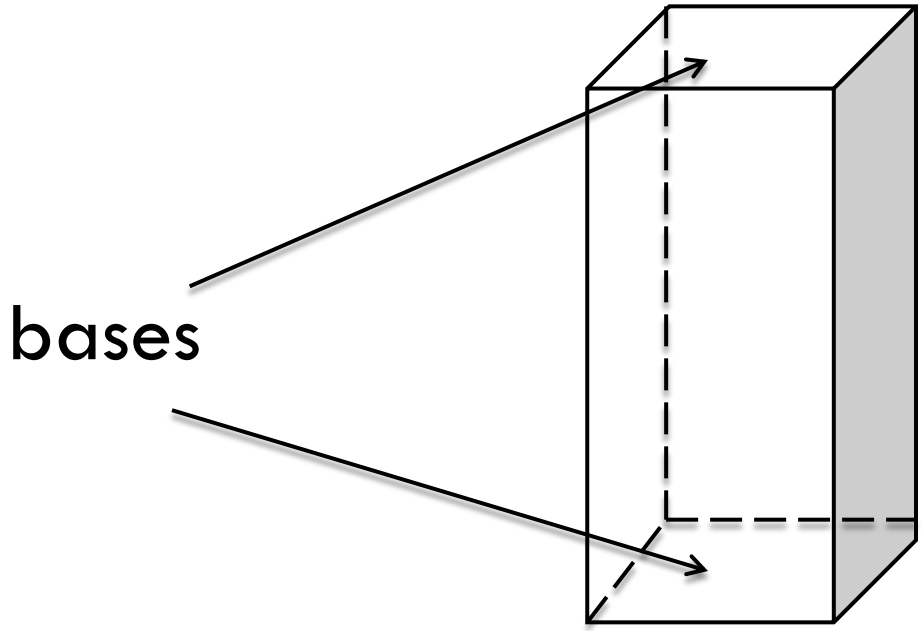
Area and Volume of Prisms

- **Content Objective**: Students will be able to identify the different types of prisms, as well as the equations for their area and volume.

- **Language Objective**: Students will be able to find the areas and volume of prisms.

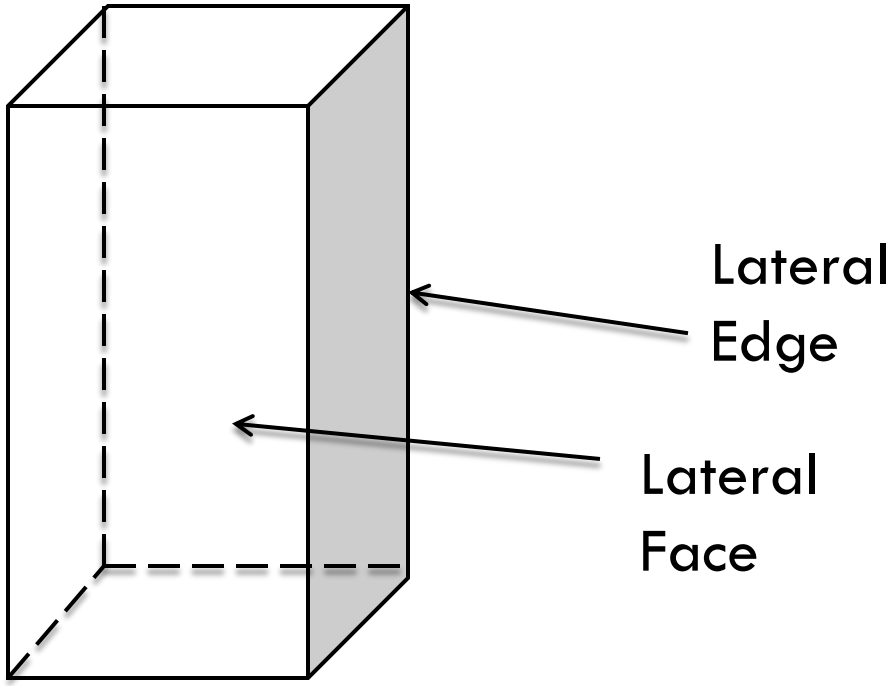
Prisms

- We will be examining 3 dimensional figures.
- The first figure we will be looking at will be the **Prisms.**
- The ends of a prism are called the **bases.**
- These bases are congruent to each other and are parallel.



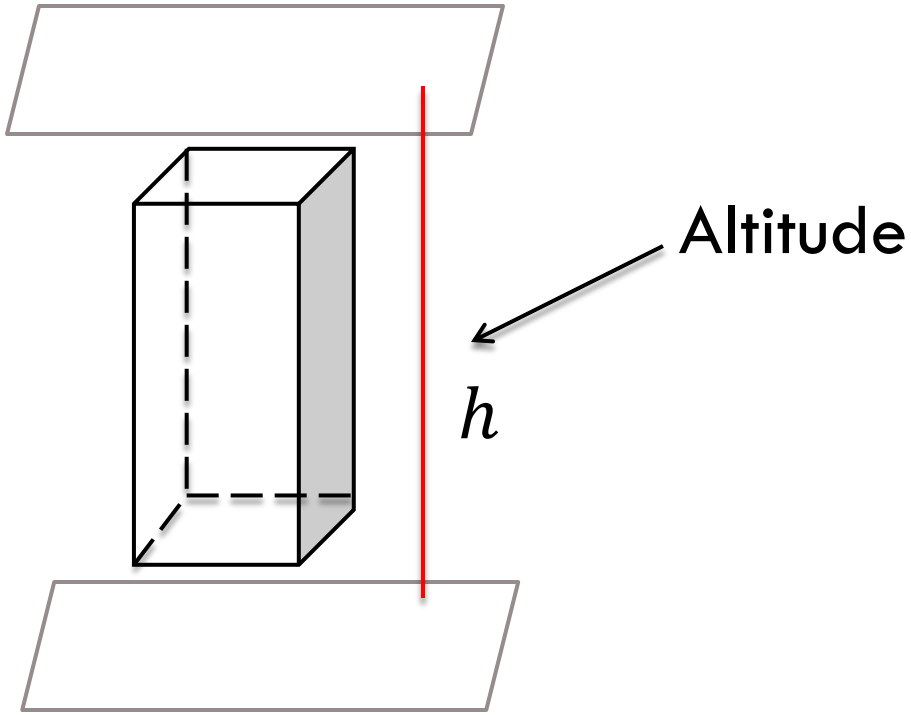
Prisms

- The faces of the prism that are not its bases are known as its **lateral faces.**
- Adjacent lateral faces intersect in parallel segments called **lateral edges.**



Prisms

- An **altitude** of a prism is a segment joining the planes that contain the bases.
- The length of the altitude is the ***height, h,*** of the prism.



What we will be Calculating.

- For prisms, we will be looking for the following values:

1.) Lateral Area: Sum of the areas of the lateral faces

2.) Total Area: The area of the entire prism

Equation: $T.A. = L.A. + 2B$

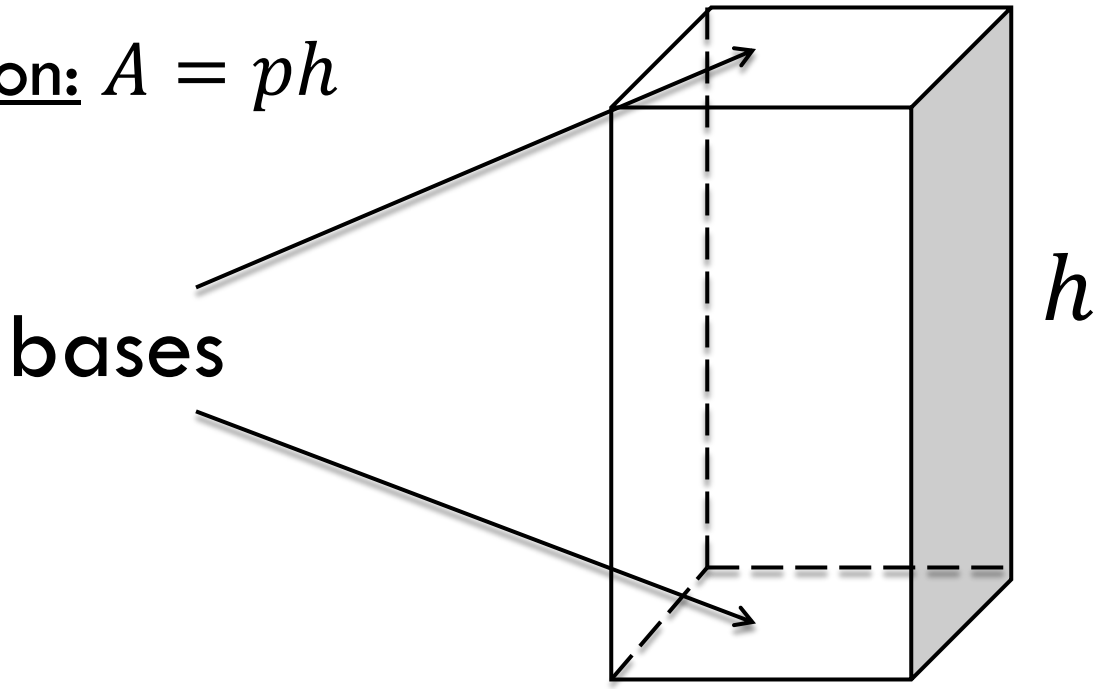
where B is the area of each base.

3.) Volume: The space that can be contained within the prism.

Area of a Prism

- **Theorem 12-1:** The lateral area of a right prism equals the perimeter of a base times the height of the prism.

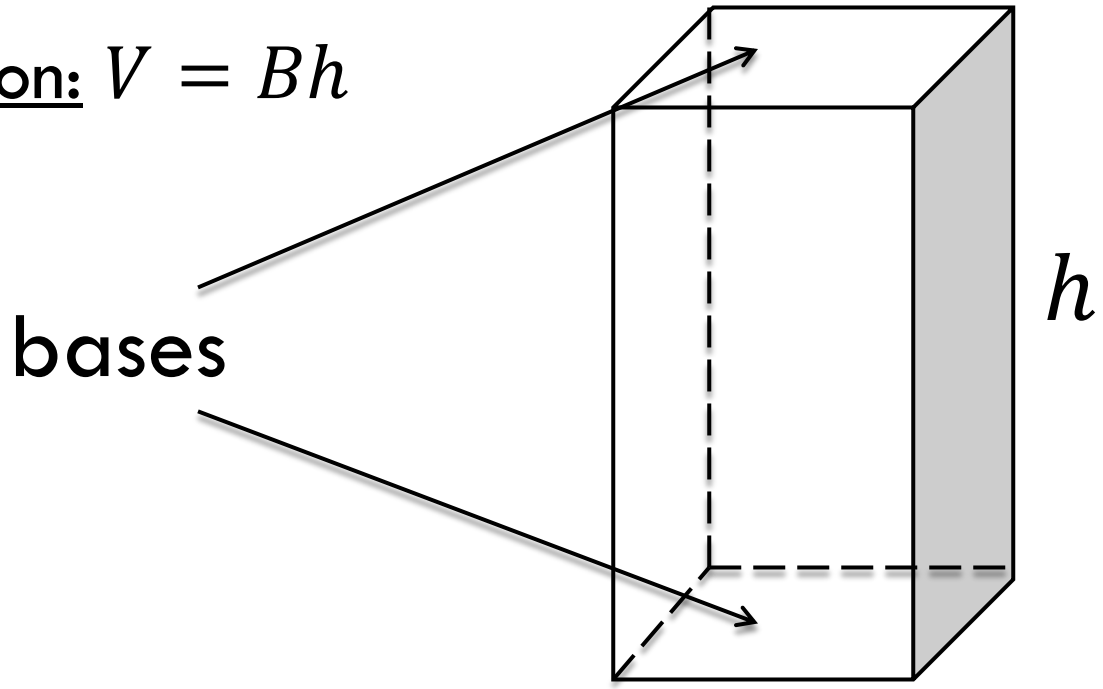
Equation: $A = ph$



Volume of a Prism

- **Theorem 12-2:** The volume of a right prism equals the area of a base times the height of the prism.

Equation: $V = Bh$

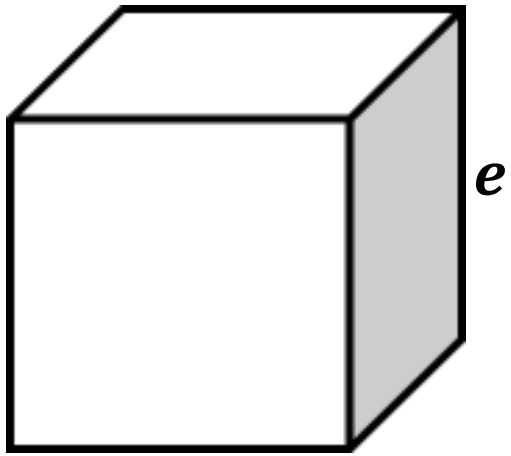


Cubes

- A rectangular prism with square faces is known as a **cube.**
- Since each face is a square, then all of its edges have equal length.
- The lateral and total areas are found using the same formulas given.
- The volume however can be simplified to:

$$V = e^3$$

where e represents a single edge



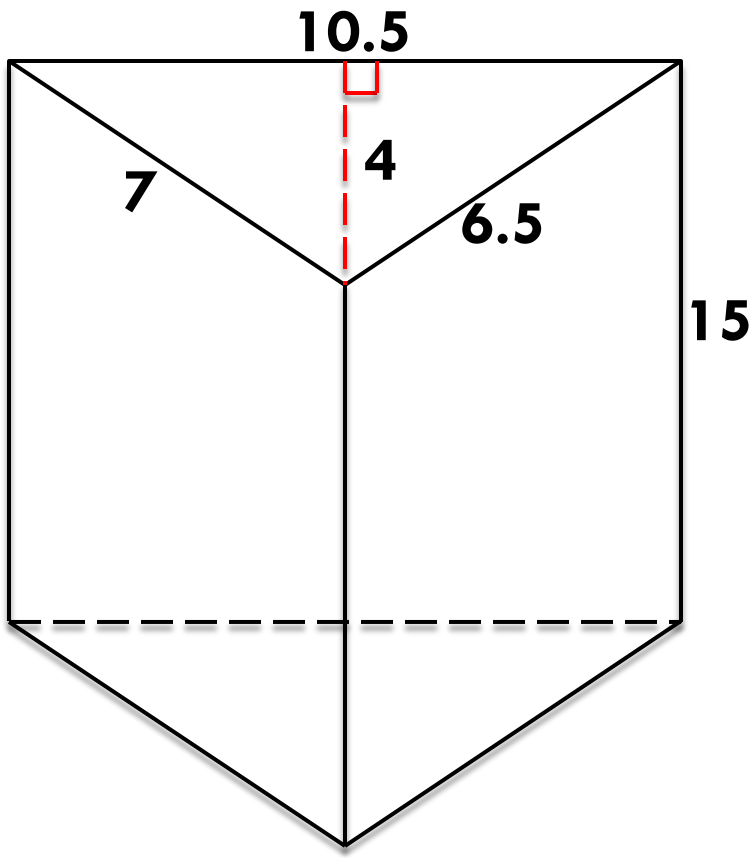
Example

- Given a right triangular prism, find the

a.) Lateral Area

b.) Total Area

c.) Volume



Example #1 Solution

Lateral Area

$$L.A. = ph$$

$$= (6.5 + 7 + 10.5) \times 15$$

$$= 24 \times 15$$

$$= \mathbf{360}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 360 + 2\left(\frac{1}{2} \times 10.5 \times 4\right)$$

$$= 360 + 42$$

$$= \mathbf{402}$$

Volume

$$V = Bh$$

$$= \left(\frac{1}{2} \times 10.5 \times 4\right) \times 15$$

$$= 21 \times 15$$

$$= \mathbf{315}$$

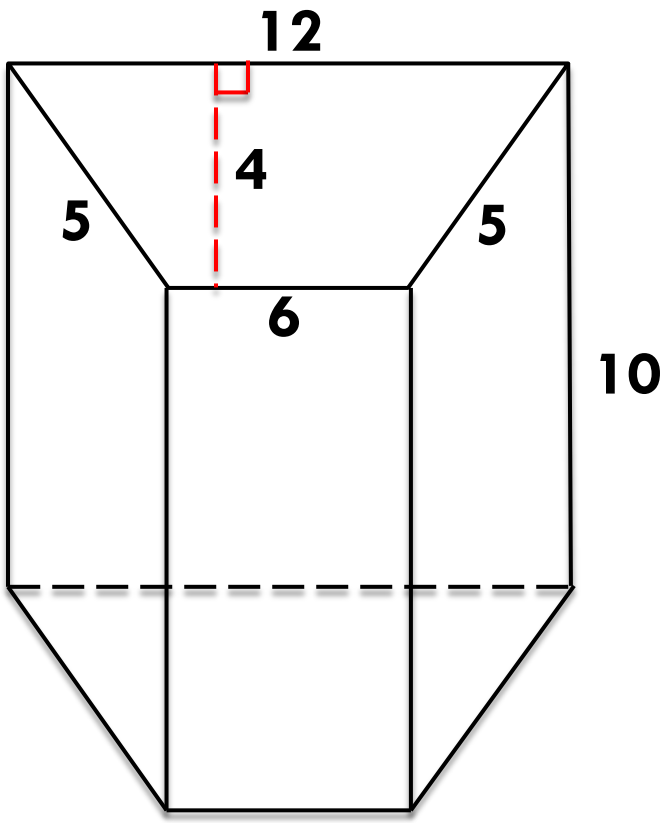
Example

- Given a right trapezoidal prism, find the

a.) Lateral Area

b.) Total Area

c.) Volume



Example #2 Solution

Lateral Area

$$L.A. = ph$$

$$= (5 + 5 + 6 + 12) \times 10$$

$$= 28 \times 10$$

$$= \mathbf{280}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 280 + 2 \times 4 \times \frac{1}{2}(6 + 12)$$

$$= 280 + 72$$

$$= \mathbf{352}$$

Volume

$$V = Bh$$

$$= 4 \times \frac{1}{2}(6 + 12) \times 10$$

$$= 36 \times 10$$

$$= \mathbf{360}$$

Group Practice

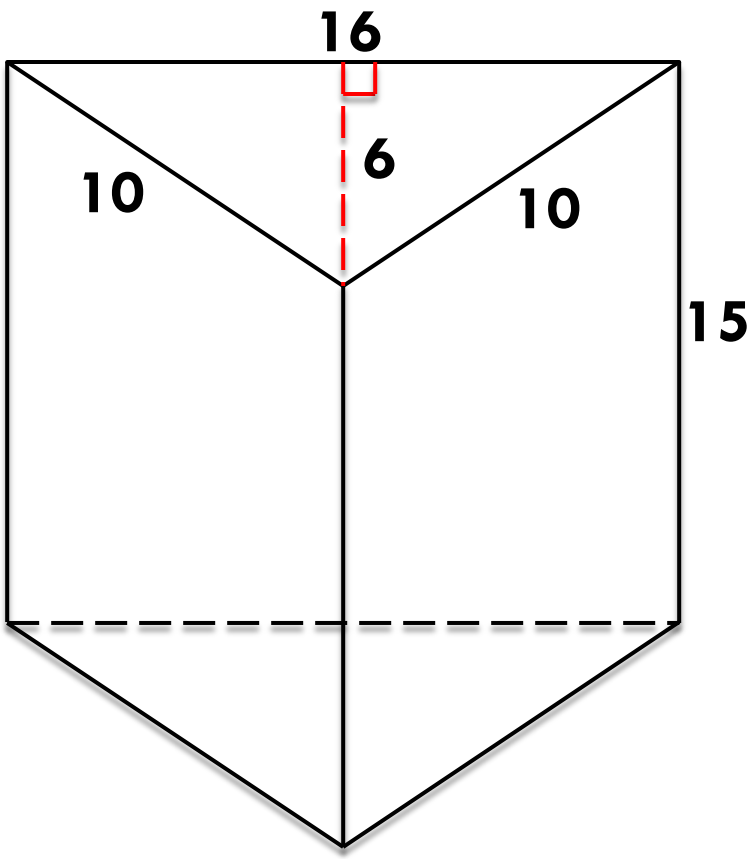
- For each of the following right prisms, find the

a.) Lateral Area

b.) Total Area

c.) Volume

1.)



Group #1 Solution

Lateral Area

$$L.A. = ph$$

$$= (16 + 10 + 10) \times 15$$

$$= 36 \times 15$$

$$= 540$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 540 + 2\left(\frac{1}{2} \times 16 \times 6\right)$$

$$= 540 + 96$$

$$= \mathbf{636}$$

Volume

$$V = Bh$$

$$= \left(\frac{1}{2} \times 16 \times 6\right) \times 15$$

$$= 48 \times 15$$

$$= \mathbf{720}$$

Group Practice

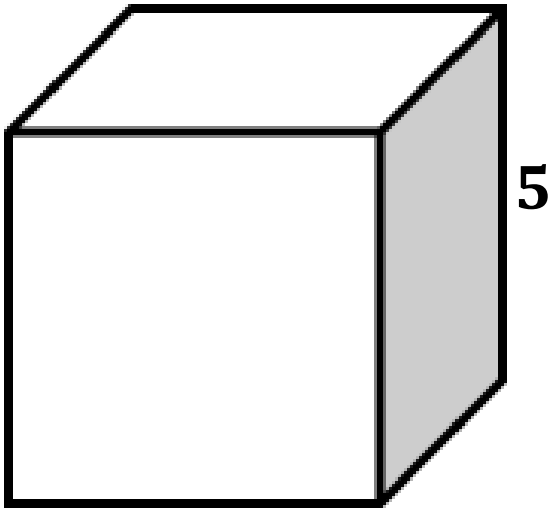
- For each of the following right prisms, find the

a.) Lateral Area

b.) Total Area

c.) Volume

2.)



Group #2 Solution

Lateral Area

$$L.A. = ph$$

$$= 4(5) \times 5$$

$$= 20 \times 5$$

$$= \mathbf{100}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 100 + 2(5^2)$$

$$= 100 + 50$$

$$= \mathbf{150}$$

Volume

$$V = Bh$$

$$= 5^2 \times 5$$

$$= 25 \times 5$$

$$= \mathbf{125}$$

Group Practice

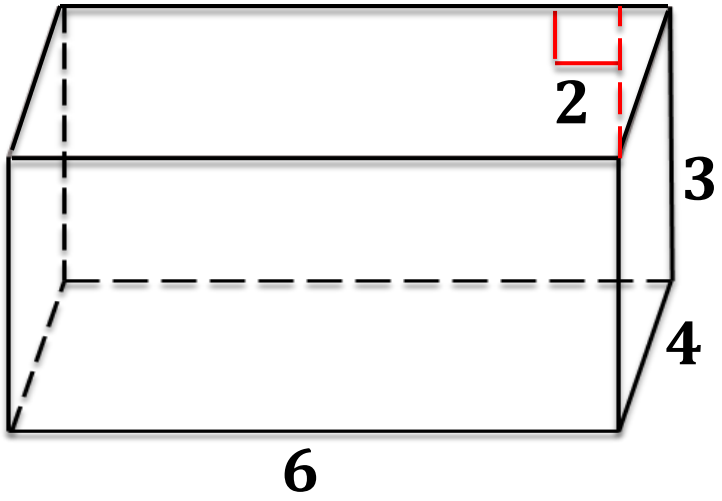
- For each of the following right prisms, find the

a.) Lateral Area

b.) Total Area

c.) Volume

3.)



Group #3 Solution

Lateral Area

$$L.A. = ph$$

$$= (6 + 6 + 4 + 4) \times 3$$

$$= 20 \times 3$$

$$= \mathbf{60}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 60 + 2(2 \times 6)$$

$$= 60 + 24$$

$$= \mathbf{84}$$

Volume

$$V = Bh$$

$$= (2 \times 6) \times 3$$

$$= 12 \times 3$$

$$= \mathbf{36}$$

Group Practice

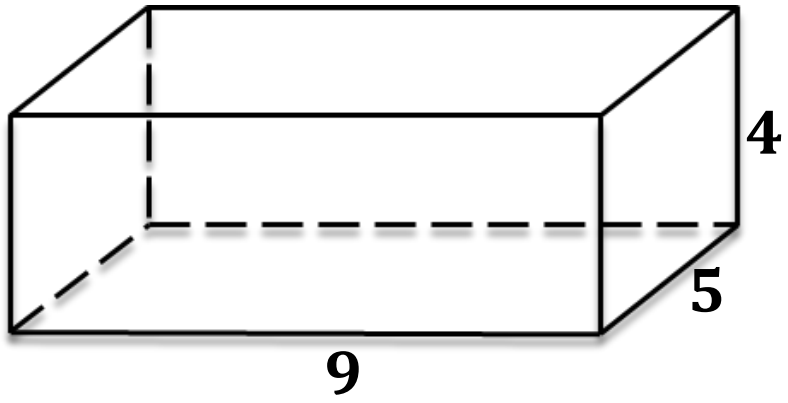
- For each of the following right prisms, find the

a.) Lateral Area

b.) Total Area

c.) Volume

4.)



Group #4 Solution

Lateral Area

$$L.A. = ph$$

$$= (9 + 9 + 5 + 5) \times 4$$

$$= 28 \times 4$$

$$= \mathbf{112}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 112 + 2(9 \times 5)$$

$$= 112 + 90$$

$$= \mathbf{202}$$

Volume

$$V = Bh$$

$$= (9 \times 5) \times 4$$

$$= 45 \times 4$$

$$= \mathbf{180}$$

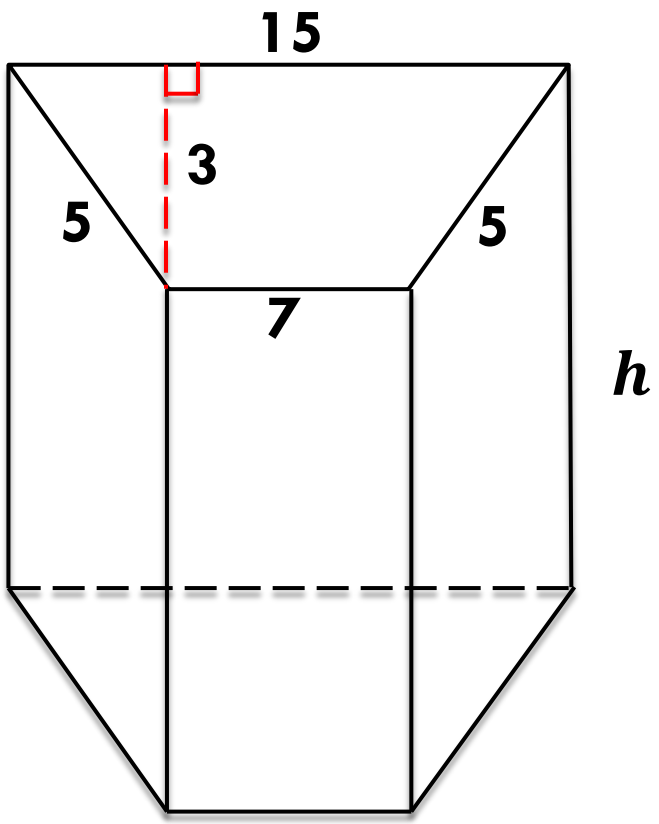
Group Practice

- For the following right prisms, you are given lateral area or the volume. First find the height, then find the remaining values.

Given:

Volume: $V = 330$

5.)



Group #5 Solution

Height

$$V = Bh$$

$$330 = 3 \times \frac{1}{2}(7 + 15) \times h$$

$$330 = 33 \times h$$

$$\boldsymbol{h = 10}$$

Lateral Area

$$L.A. = ph$$

$$= (15 + 7 + 5 + 5) \times 10$$

$$= 32 \times 10$$

$$= \mathbf{320}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 320 + 2 \times 3 \times \frac{1}{2}(7 + 15)$$

$$= 320 + 66$$

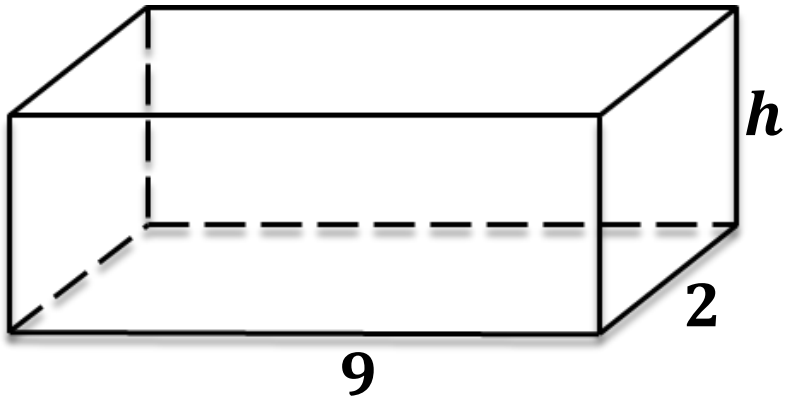
$$= \mathbf{386}$$

Group Practice

- For the following right prisms, you are given lateral area or the volume. First find the height, then find the remaining values. 6.)

Given:

Lateral Area: $L.A. = 66$



Group #6 Solution

Lateral Area

$$L.A. = ph$$

$$66 = (9 + 9 + 2 + 2) \times h$$

$$66 = 22 \times \mathrm{h}$$

$$\boldsymbol{h = 3}$$

Total Area

$$T.A. = L.A. + 2B$$

$$= 66 + 2(9 \times 2)$$

$$= 66 + 36$$

$$= \mathbf{102}$$

Volume

$$V = Bh$$

$$= (9 \times 2) \times 3$$

$$= 18 \times 3$$

$$= \mathbf{54}$$