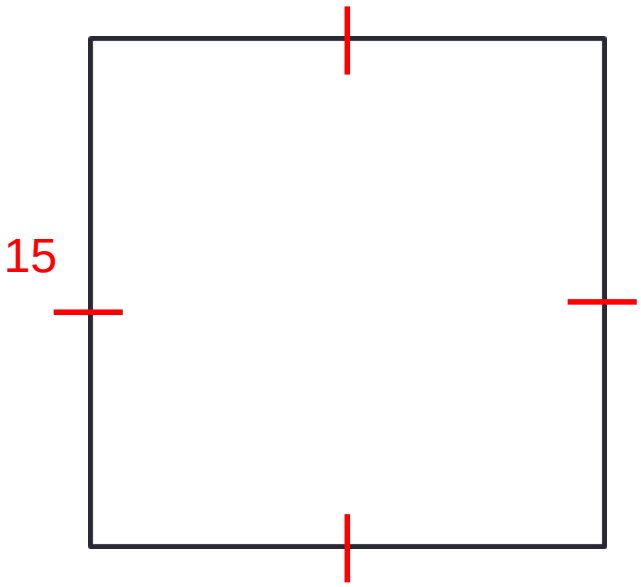
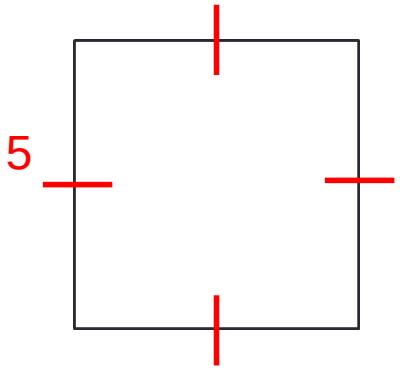
GEOMETRY UNIT 11

12-5: Area and Volumes of Similar Solids

Warm-up

- Look at the following shapes and answer the following questions:
  - What do you notice about the shapes?
  - How are they alike?
  - How are they different?
  - What is the key word that links these shapes together.
  - It is that they are ___Similar___.





Area and Volumes of Similar Solids

- **Content Objective**: Students will be able to identify ratios between the values of similar solids.

- **Language Objective**: Students will be able to find missing values using proportions between similar solids.

Recall: Similar Polygons

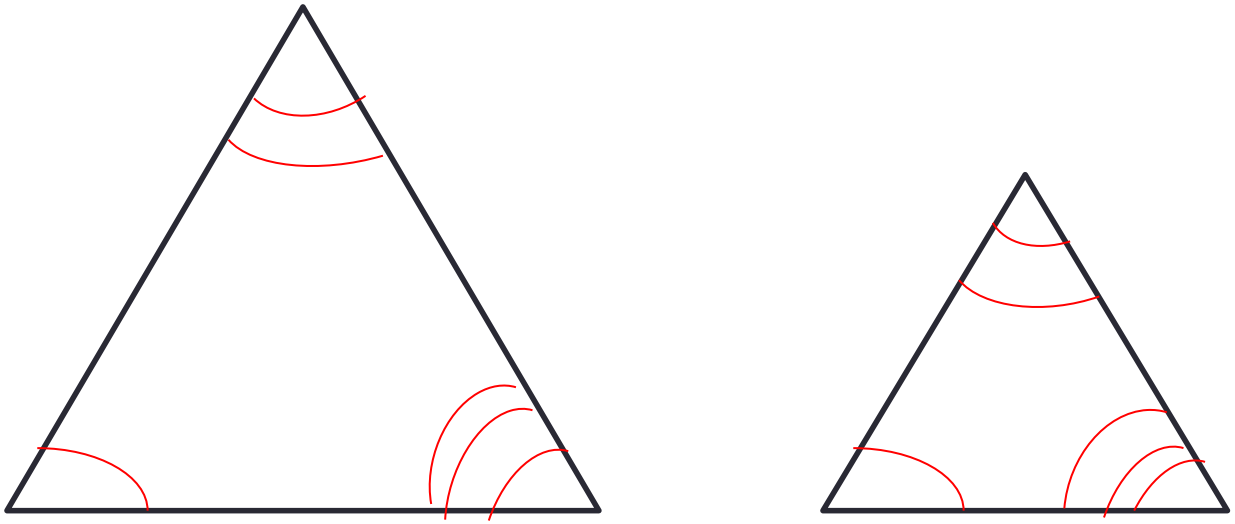
- Two polygons are **similar** if their vertices can be paired so that:

1.) Corresponding angles are congruent.

2.) Corresponding sides are in proportion.

(i.e. Their side lengths have the same ratio.)

Ex:



Similar Solids

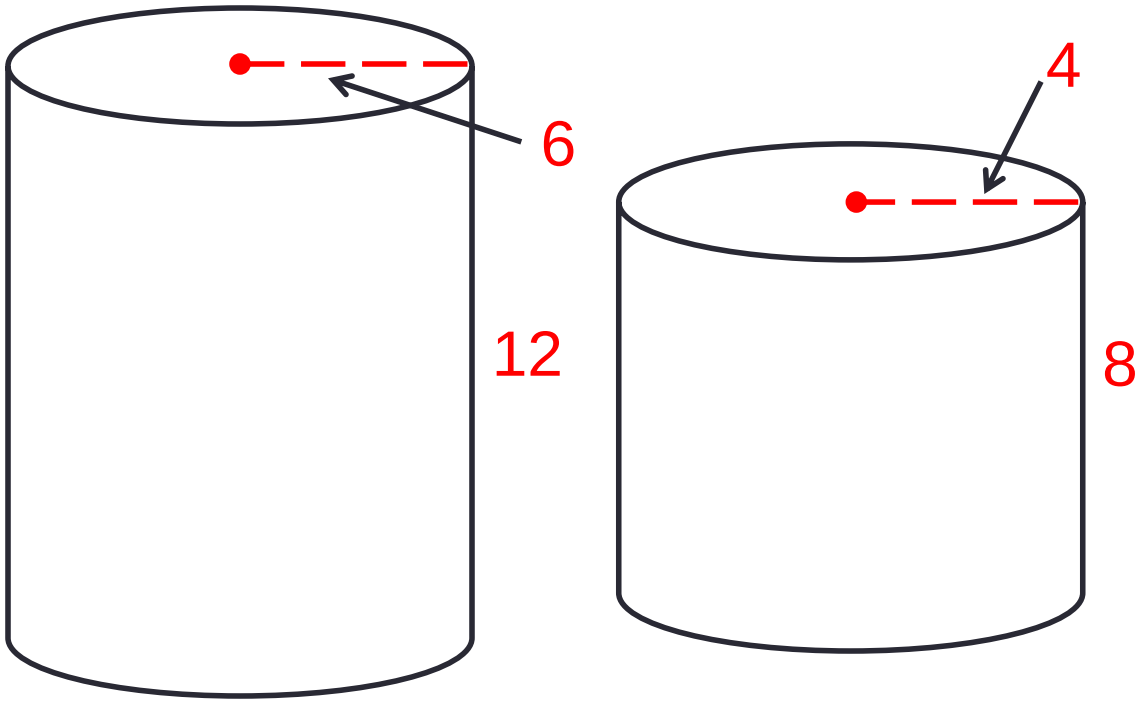
- **Similar Solids** are solids that have the same shape but not necessarily the same size.

- To determine if two solids are similar, you must check
  - That their bases are **similar**
  - That corresponding lengths are **proportional**

- If the solids are similar, we will be able to identify a ratio between their corresponding parts, known as the **Scale Factor**.

Checking for Similarity

Examine the following Cylinders:



- The bases are similar (all circles are similar).
- As for the lengths, they are proportional, because

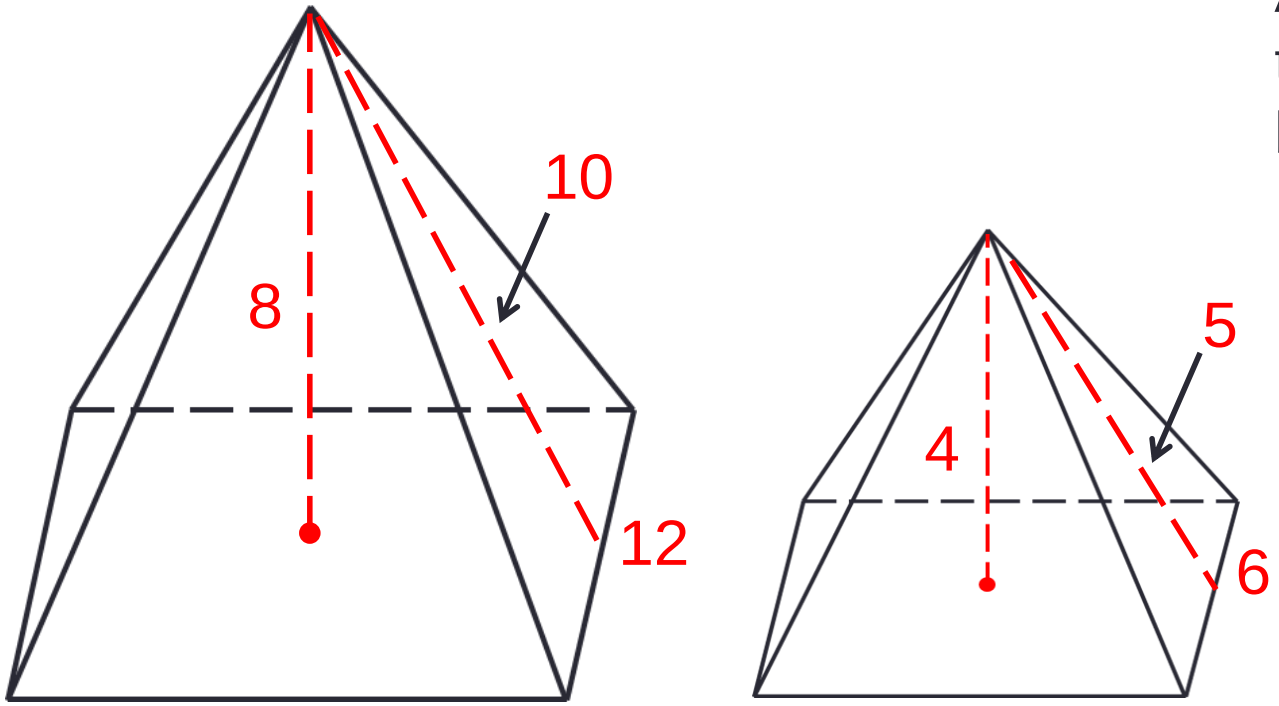
$$\frac{6}{4} = \frac{12}{8}$$

$$\frac{\mathbf{3}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}}$$

Therefore, the two cylinders are similar with a scale factor of $\frac{\mathbf{3}}{\mathbf{2}}$

Examples of Similar Solids

Examine the following Cylinders:



- The bases are similar (why?).
- As for the lengths, they are proportional, because

$$\frac{8}{4} = \frac{12}{6} = \frac{10}{5}$$

$$\mathbf{2 = 2 = 2}$$

Therefore, the two Pyramids are similar with a scale factor of $\mathbf{2}$

Examining Similarity

- Depending on what values we are comparing the scale factor between similar solids may change.
- Find the scale factor between the values given in each of the first two columns. Identify how these scale factors relate to the original scale factor given.

	Pyramid I	Pyramid II
Scale Factor:		$2/1$
Base Perimeter:	12	6
Lateral Area:	240	60
Volume:	384	48

Comparing the Scale Factors

- Now, determine the scale factor between the values given from each shape, then compare them to the original scale factor between the shapes.

	Base Perimeter	Lateral Area	Volume
$\frac{I}{II}$	$\frac{12}{6} = \frac{2}{1}$	$\frac{240}{60} = \frac{4}{1}$	$\frac{384}{48} = \frac{8}{1}$
Compared to Scale Factor:	$\frac{2}{1} = \frac{2}{1}$	$\frac{4}{1} = \frac{2^2}{1^2}$	$\frac{8}{1} = \frac{2^3}{1^3}$

Can you see the relationship between the original scale factor and the scale factors for the base perimeter, lateral area, and volume?

Examining Similarity

- Now we are going to see how this relationship affects the area and volume of similar solids.
- Find the scale factor between the values given in each of the first two columns. Identify how these scale factors relate to the original scale factor given.

	Cylinder I	Cylinder II
Scale Factor:	$3/2$	
Base Circumference:	12π	8π
Lateral Area:	144π	64π
Volume:	1728π	512π

Comparing the Scale Factors

- Now, determine the scale factor between the values given from each shape, then compare them to the original scale factor between the shapes.

	Base Circumference	Lateral Area	Volume
$\frac{I}{II}$	$\frac{12\pi}{8\pi} = \frac{3}{2}$	$\frac{144\pi}{64\pi} = \frac{9}{4}$	$\frac{384\pi}{48\pi} = \frac{27}{8}$
Compared to Scale Factor:	$\frac{3}{2} = \frac{3}{2}$	$\frac{9}{4} = \frac{3^2}{2^2}$	$\frac{27}{8} = \frac{3^3}{2^3}$

Can you see the relationship between the original scale factor and the scale factors for the base circumference, lateral area, and volume?

Theorem for Similar Solids

Theorem 12-11: If the scale factor of two similar solids is $\boldsymbol{a}:\boldsymbol{b}$, then

(1) The ratio of corresponding perimeters is $\boldsymbol{a}:\boldsymbol{b}$

(2) The ratio of the base areas, of the lateral area, and of the total areas is $\boldsymbol{a^2}:\boldsymbol{b^2}$

(3) The ratio of the volumes is $\boldsymbol{a^3}:\boldsymbol{b^3}$

Practice

1.) Given the following measurements for similar solids, identify the reduced ratio for each of the following.

Given height 2 and height 5

(a.) Scale Factor $\frac{2}{5}$

(b.) Total Area $\frac{2^2}{5^2} = \frac{4}{25}$

Practice

- 2.) Given the following measurements for similar solids, identify the reduced ratio for each of the following.

Given areas 4π and 12π.

(a.) Scale Factor $\frac{4\pi}{12\pi} = \frac{1}{3}$

(b.) Volume $\frac{1^3}{3^3} = \frac{1}{27}$

Practice

- 3.) The following solids are similar. Use the given information to solve for the value:
- The scale factor of solid A : solid B is 3:4.
- If solid A has a circumference of 18, calculate the circumference of solid B.

Solution: From the Theorem, we have

$$\frac{Circ.A}{Circ.B} = \frac{A}{B}$$

$$\frac{18}{x} = \frac{3}{4}$$

$$3x = 72$$

$$\boldsymbol{x = 24}$$

Practice

- 4.) The following solids are similar. Use the given information to solve for the value:
- The scale factor of solid C : solid D is 6:5.
- If solid C has a base area of 108, calculate the base area of solid D.

Solution: From the Theorem, we have

$$\frac{Base\ C}{Base\ D} = \frac{C^2}{D^2} = \frac{6^2}{5^2}$$

$$\frac{108}{x} = \frac{36}{25}$$

$$36x = 27000$$

$$\boldsymbol{x = 75}$$

Group Practice

1.) Given the following measurements for similar solids, identify the reduced ratio for each of the following.

Given height 4 and height 7

(a.) Scale Factor $\frac{4}{7}$

(b.) Total Area $\frac{4^2}{7^2} = \frac{16}{49}$

Group Practice

- 2.) Given the following measurements for similar solids, identify the reduced ratio for each of the following.

Given areas 3π and 5π.

(a.) Scale Factor $\frac{3\pi}{5\pi} = \frac{3}{5}$

(b.) Volume $\frac{3^3}{5^3} = \frac{27}{125}$

Group Practice

- 3.) The following solids are similar. Use the given information to solve for the value:
- The scale factor of solid A : solid B is 7:8.
- If solid A has a perimeter of 18, calculate the perimeter of solid B.

Solution: From the Thm 12-11, we have

$$\frac{Per.A}{Per.B} = \frac{A}{B}$$

$$\frac{35}{x} = \frac{7}{8}$$

$$7x = 280$$

$$\boldsymbol{x = 40}$$

Group Practice

- 4.) The following solids are similar. Use the given information to solve for the value:
- The scale factor of solid C : solid D is 5:1.
- If solid C has a lateral area of 108, calculate the lateral area of solid D.

Solution: From the Thm 12-11, we have

$$\frac{L.A.\ of\ C}{L.A.\ of\ D} = \frac{C^2}{D^2} = \frac{5^2}{1^2}$$

$$\frac{100}{x} = \frac{25}{1}$$

$$25x = 100$$

$$\boldsymbol{x = 4}$$