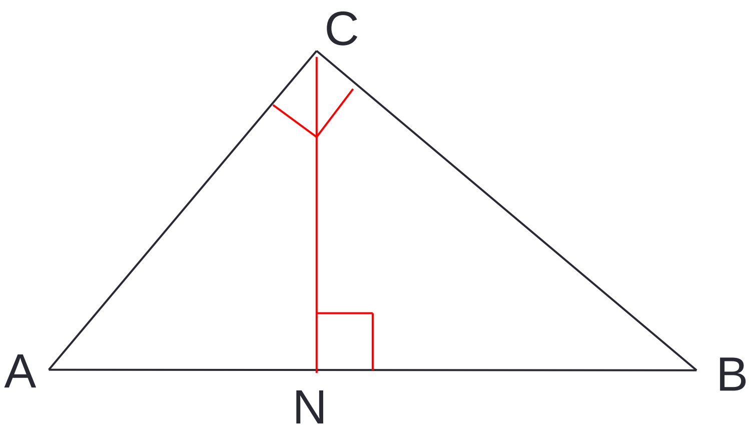
GEOMETRY UNIT 8

8-1: Similarity in Right Triangles

Warm-up

- Prove the following similarities (Using one of the postulates/theorems)



1.) $\Delta ACB \sim \Delta ANC$ By AA~

$< ACB \cong < ANC$ (Why?)

$< A \cong < A$ (How?)

2.) $\Delta ACB \sim \Delta CNB$ By AA~

$< ACB \cong < CNB$ (Why?)

$< B \cong < B$ (How?)

3.) $\Delta ANC \sim \Delta CNB$ By substitution form 1) and 2)

Similarity in Right Triangles

- **Content Objective:** Students will be able to find the geometric mean of two numbers and of the sides of triangles.

- **Language Objective:** Students will be able write simplified expressions using radicals.

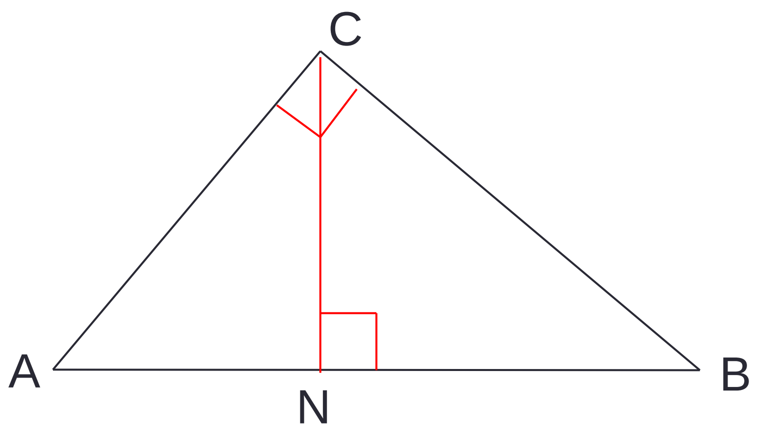
Triangles Similarity Theorem

- **<u>Theorem 8-1:</u>** If the altitude of a right triangle is drawn on the hypotenuse, then the two triangles formed are similar to the original triangle and to each other.

Given: ΔABC with rt. $< ACB$

altitude $\overline{CN}$

Prove: $\Delta ACB \sim \Delta ANC \sim \Delta CNB$



Geometric Mean

- Recall that in the proportions $\frac{a}{x} = \frac{y}{b}$, the terms in red (x and y) are called the **Means.**
- If *a*, *b*, and *x* are positive numbers and $\frac{a}{x} = \frac{x}{b}$, then x is called the **Geometric Mean.**
- If you solve this proportion for x…
- Then $x = \sqrt{\boldsymbol{ab}}$
- Try it – Find the Geometric mean for these numbers:

2 and 18

$$x = \sqrt{2 * 18}$$
$$= \sqrt{36}$$
$$= 6$$

3 and 27

$$x = \sqrt{3 * 27}$$
$$= \sqrt{81}$$
$$= 9$$

22 and 55

$$x = \sqrt{22 * 55}$$
$$= \sqrt{2 * 11 * 5 * 11}$$
$$= 11\sqrt{10}$$

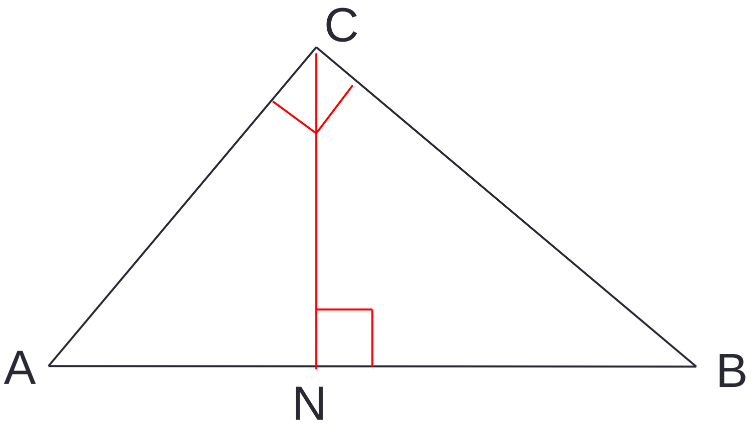
Corollaries

- **<u>Corollary 1:</u>** When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.

Given: ΔABC with rt. $< ACB$

altitude $\overline{CN}$

Prove: $\dfrac{AN}{CN} = \dfrac{CN}{BN}$



Corollaries

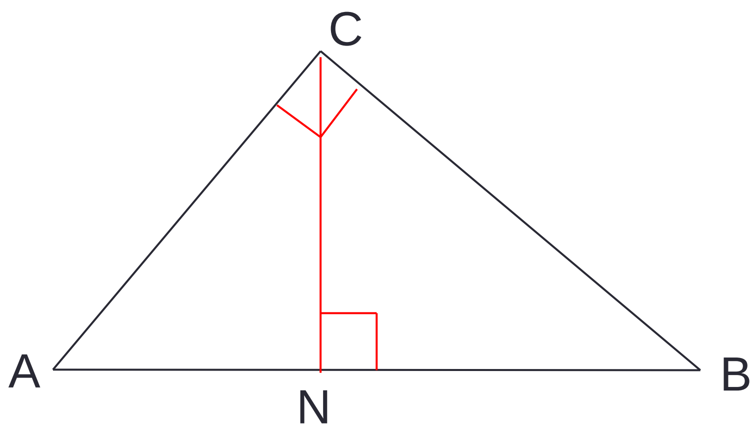
- **Corollary 2:** When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

Given: ΔABC with rt. $< ACB$

altitude $\overline{CN}$

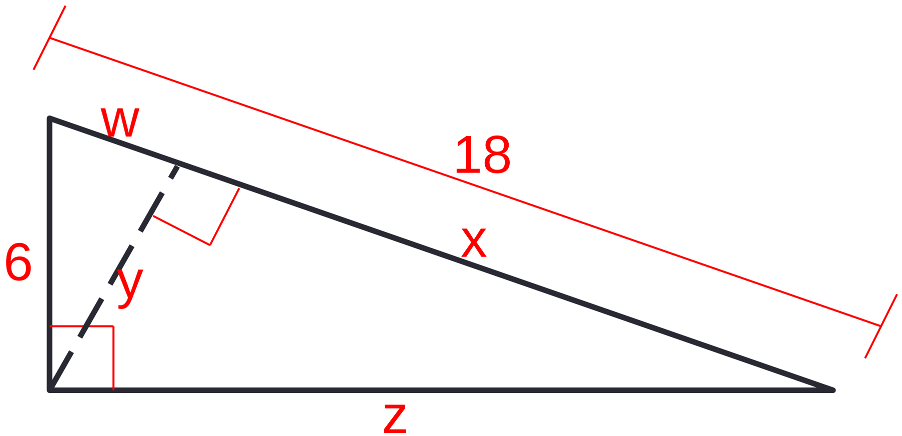
Prove: 1.) $\frac{AB}{AC} = \frac{AC}{AN}$

2.) $\frac{AB}{BC} = \frac{BC}{BN}$



Geometric Mean Examples

- Use the proportions given in corollaries 1 and 2 to find the values of w, x, y, and z.



For w:

$$\frac{18}{6} = \frac{6}{w} \quad \text{(Why?)}$$

$$18 = 36w$$

$$w = 2$$

For x: $x = 18 - 2 = 16$

For y:

$$\frac{16}{y} = \frac{y}{2} \quad \text{(Why?)}$$

$$y^2 = 32$$

$$y = \sqrt{32} = \sqrt{16 * 2}$$

$$y = 4\sqrt{2}$$

For z:

$$\frac{18}{z} = \frac{z}{16} \quad \text{(Why?)}$$

$$z^2 = 18 * 16$$

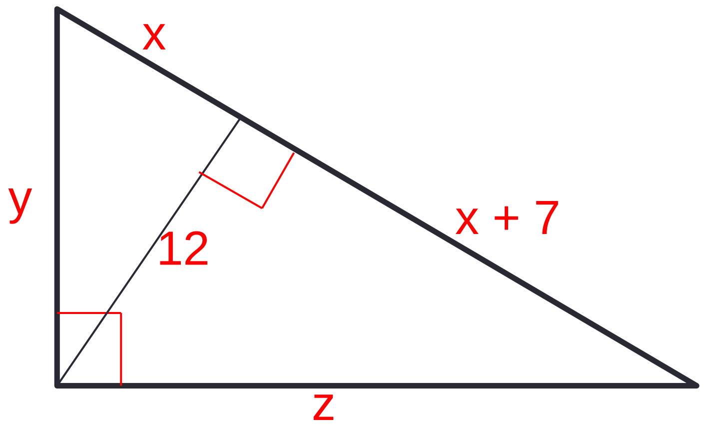
$$z = \sqrt{18 * 16}$$

$$z = \sqrt{2 * 9 * 16}$$

$$z = 3 * 4\sqrt{2} = 12\sqrt{2}$$

Geometric Mean Examples

- Use the proportions given in corollaries 1 and 2 to find the values of x, y, and z.



For x:

$$\frac{x+7}{12} = \frac{12}{x}$$

$$144 = x^2 + 7x$$

$$x^2 + 7x - 144 = 0$$

$$(x+16)(x-9) = 0$$

$x + 16 = 0$ and $x - 9 = 0$

$x = 9$ and

$$x = -16$$

For y:

$$\frac{25}{y} = \frac{y}{9}$$

$$y^2 = 225$$

$$y = \sqrt{225}$$

$$y = 15$$

For z:

$$\frac{25}{z} = \frac{z}{16}$$

$$z^2 = 400$$

$$z = \sqrt{400}$$

$$z = 20$$