GEOMETRY UNIT 8

8-4: SPECIAL RIGHT TRIANGLES

WARM-UP

Find the value of x in these right triangles.

1.)

x

16

20

2.)

x

8   15

3.)

9

x

9
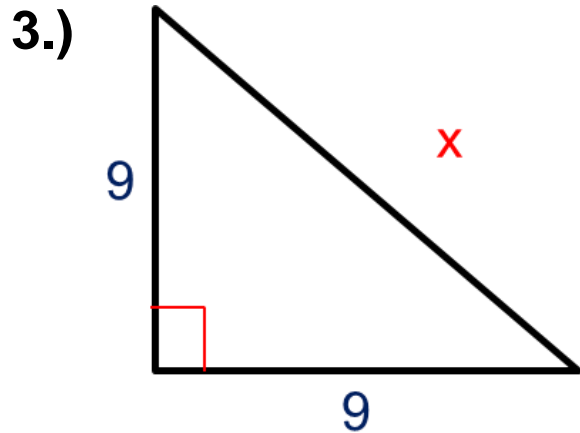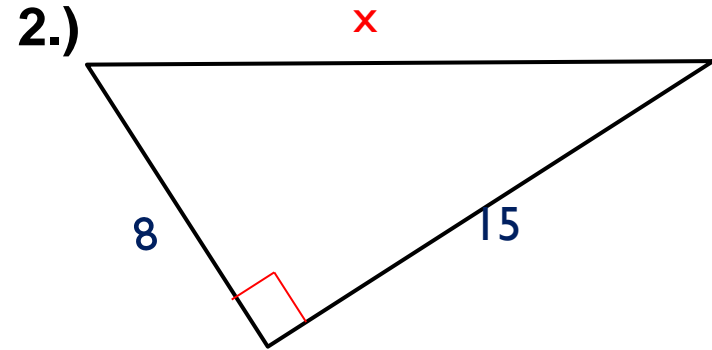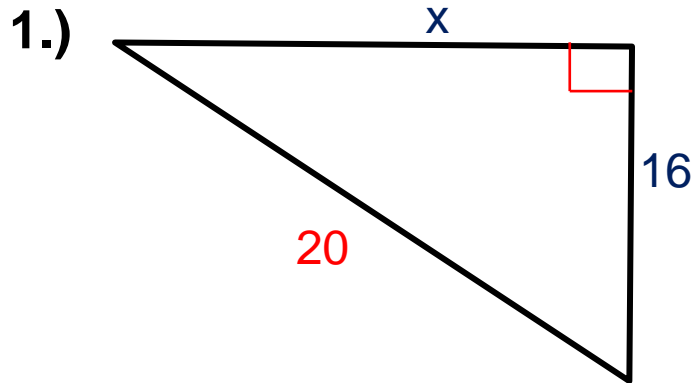
SPECIAL RIGHT TRIANGLES

Content Objective: Students will be able to solve for missing side lengths in special right triangles.
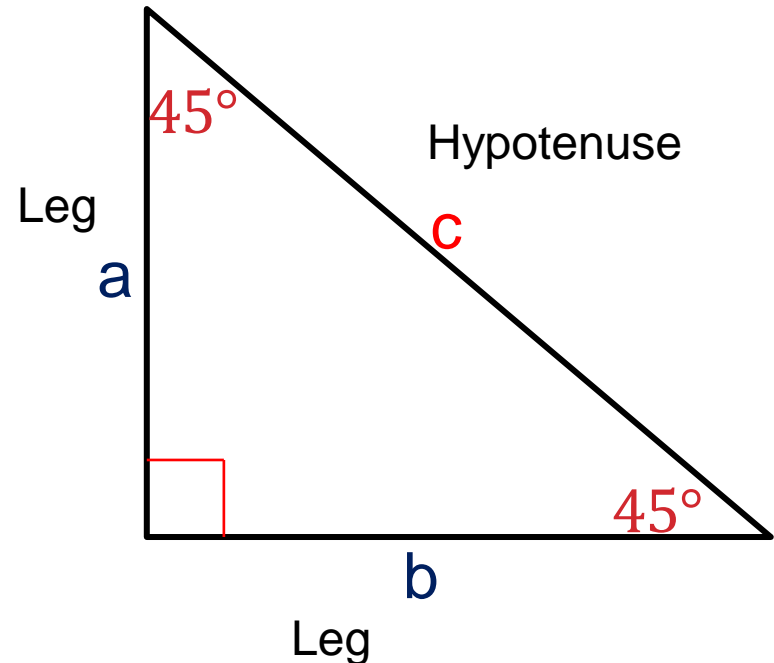
Language Objective: Students will be able to name the special right triangles, as well as label their specific parts.

$45° - 45° - 90°$ **THEOREM**

Theorem 8-6: In a $45° - 45° - 90°$ right triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.
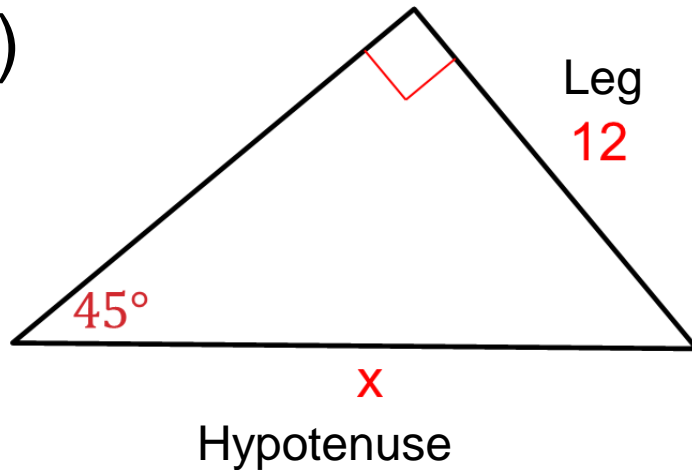
Hypotenuse

$$c = a\sqrt{2} \quad \text{or} \quad c = b\sqrt{2}$$

Leg a

45°

Hypotenuse

c

45°

b

Leg

$45° - 45° - 90°$ EXAMPLES

Find the value of x.

1.)

Leg
12

45°

x

Hypotenuse

Solution:

$$x = 12 * \sqrt{2}$$

$$x = 12\sqrt{2}$$

2.)

Leg
x

Hypotenuse

8

45°
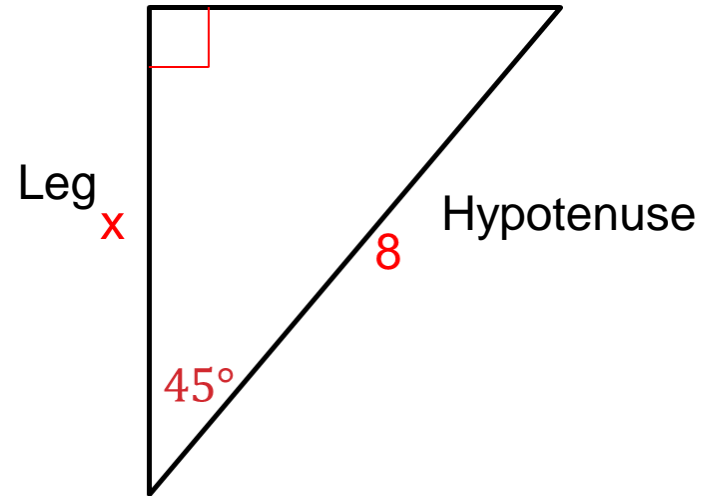
Solution:

$$8 = x\sqrt{2}$$

$$x = \frac{8}{\sqrt{2}}$$

$$x = \frac{8}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2}$$

$$x = 4\sqrt{2}$$
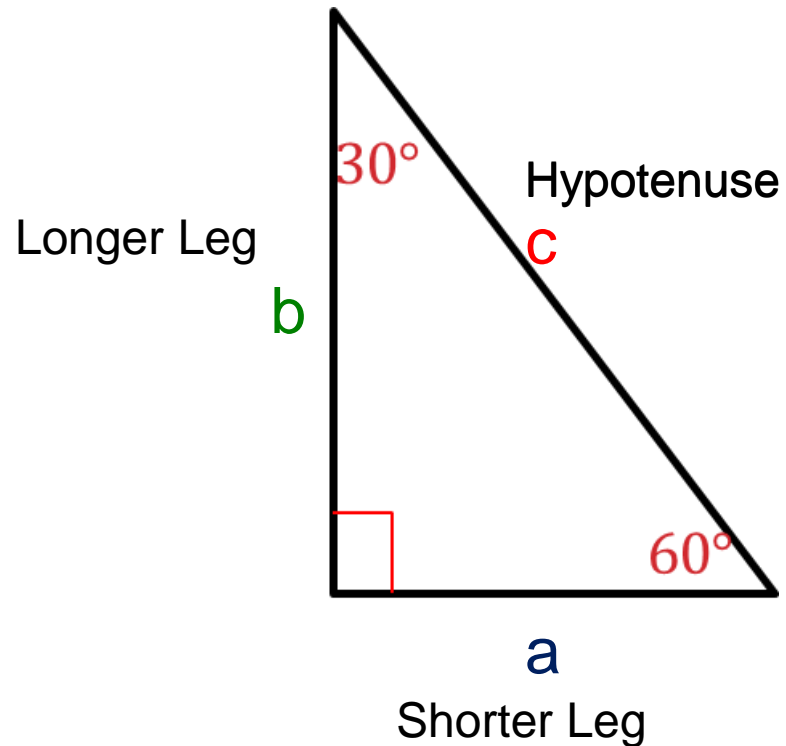
$30° − 60° − 90°$ **THEOREM**

Theorem 8-7: In a $30° − 60° − 90°$ right triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as a shorter leg.

Hypotenuse

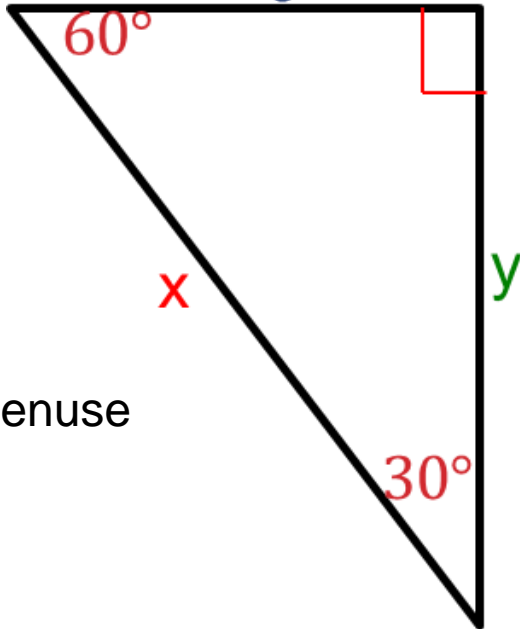$$c = 2a$$

Longer Leg

$$b = a\sqrt{3}$$

30°

Longer Leg

b

Hypotenuse

c

60°

a

Shorter Leg

$30° − 60° − 90°$ EXAMPLES

3.) Find the values of x and y

Shorter Leg

6

60°

x

Hypotenuse

y

Longer Leg

30°

For x:

$$x = 6 * 2$$

$$x = 12$$

For y:

$$y = 6 * \sqrt{3}$$

$$y = 6\sqrt{3}$$

$30° − 60° − 90°$ EXAMPLES

4.) Find the values of x and y

Shorter Leg

Longer Leg

8

x

60°   30°

y
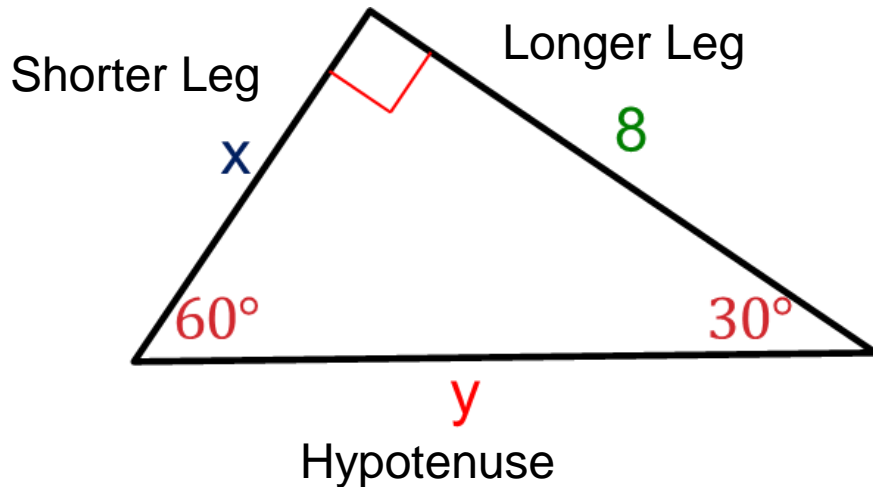
Hypotenuse

<u>For x:</u>

$$8 = x\sqrt{3}$$

$$x = \frac{8}{\sqrt{3}}$$

$$x = \frac{8}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$
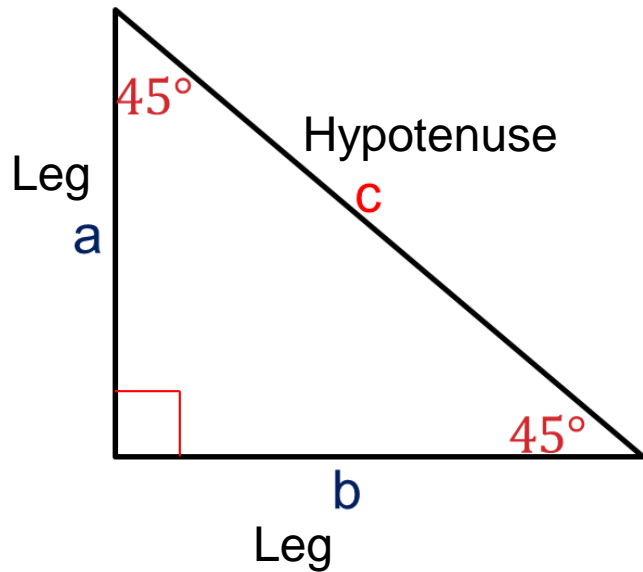
<u>For y:</u>

$$y = x * 2$$

$$y = 2 * \frac{8\sqrt{3}}{3}$$

$$y = \frac{16\sqrt{3}}{3}$$

WARM-UP: REVIEWING THE TRIANGLES

Label the parts of each triangle, then give the equations associated with them.
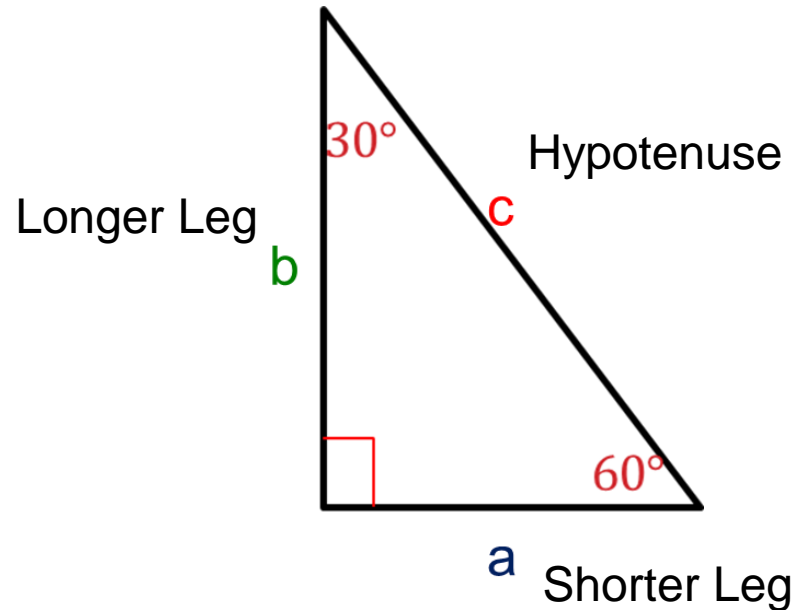
$45° - 45° - 90°$

Leg
a

$45°$

Hypotenuse
c

$45°$

b
Leg

$30° - 60° - 90°$

Longer Leg
b

$30°$

Hypotenuse
c

$60°$

a
Shorter Leg

Equation(s):

$$Hypotenuse\ (c) = Leg\ (a\ or\ b) * \sqrt{2}$$

Equation(s):

$$Hypotenuse\ (c) = Shorter\ Leg\ (a) * 2$$

$$Longer\ Leg\ (b) = Shorter\ leg\ (a) * \sqrt{3}$$

RATIO OF SIDES

In Closing, the sides of each of the special right triangles can be put into this form

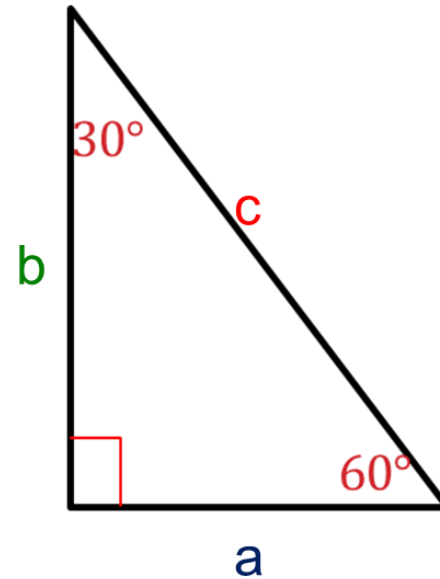$\underline{45° - 45° - 90°}$
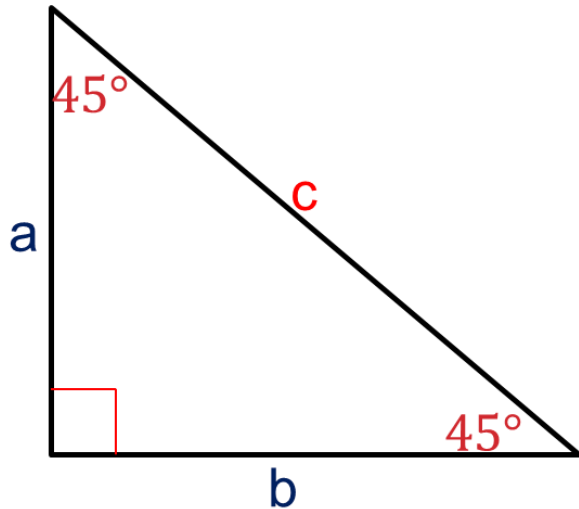
$\underline{30° - 60° - 90°}$
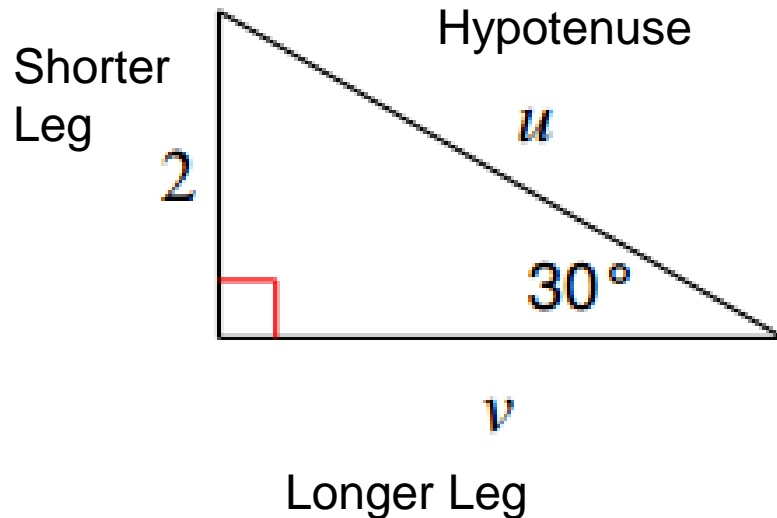

Side Ratio:

$$a:b:c \rightarrow 1:1:\sqrt{2}$$

Side Ratio:

$$a:b:c \rightarrow 1:\sqrt{3}:2$$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

1.

Shorter Leg

Hypotenuse

u

2

$30°$

v

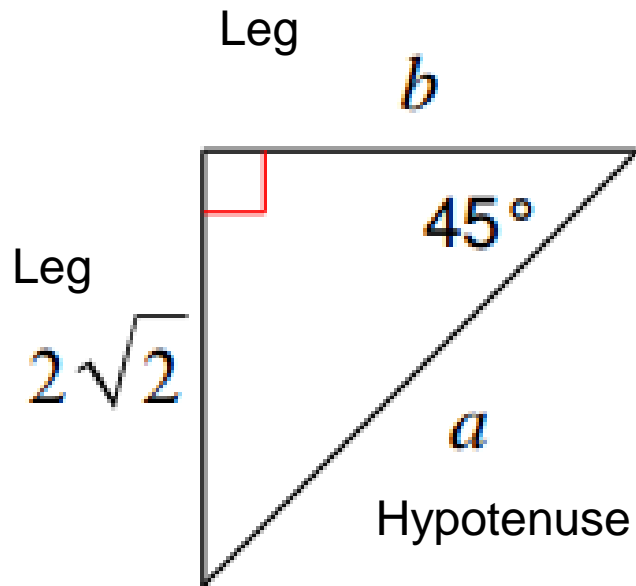Longer Leg

For u:

$$u = 2 * 2$$
$$u = 4$$

For v:

$$v = \sqrt{3} * 2$$
$$v = 2\sqrt{3}$$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

2.

Leg
b

45°

Leg
$2\sqrt{2}$

a

Hypotenuse

For b:
Legs are $\cong$ in a $45° - 45° - 90°$ Triangle.
Thus, $\qquad \boldsymbol{b = 2\sqrt{2}}$

For a:
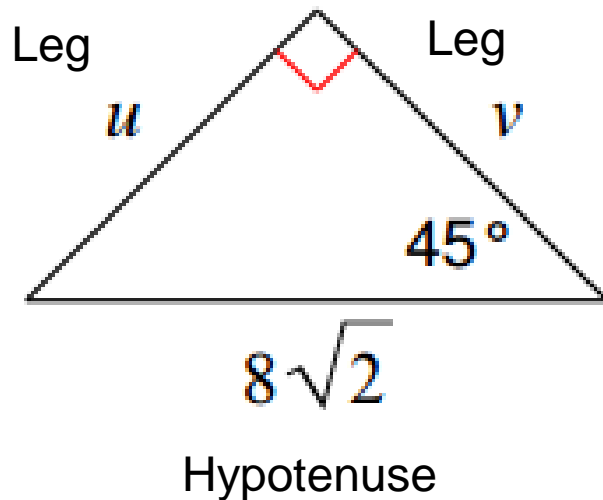
$$a = \sqrt{2} * 2\sqrt{2}$$
$$a = 2 * \sqrt{2} * \sqrt{2}$$
$$a = 2 * 2 = \boldsymbol{4}$$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

3.

Leg u

Leg v

$45°$

$8\sqrt{2}$

Hypotenuse

For v:

$$8\sqrt{2} = \sqrt{2} * v$$

$$v = \frac{8\sqrt{2}}{\sqrt{2}}$$

$$\boldsymbol{v = 8}$$

For u:

Legs are $\cong$ in a $45° - 45° - 90°$ Triangle.

Thus, $\qquad \boldsymbol{u = 8}$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

4.

Longer Leg

$11\sqrt{3}$

$30°$

Shorter Leg

b

a
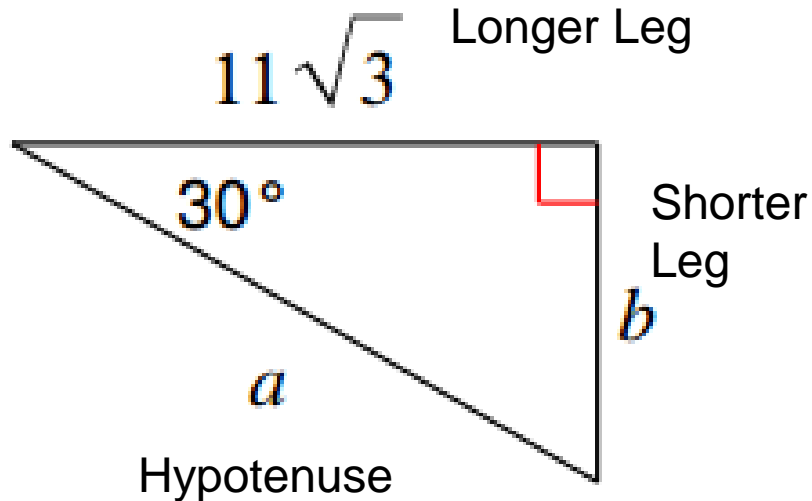
Hypotenuse

<u>For b:</u>

$$11\sqrt{3} = \sqrt{3} * b$$

$$b = \frac{11\sqrt{3}}{\sqrt{3}}$$

$$v = \mathbf{11}$$

<u>For a:</u>

$$a = 2 * 11$$

$$a = \mathbf{22}$$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

5.

Hypotenuse

$8\sqrt{5}$

$60°$

y

x

Shorter Leg

Longer Leg

For y:

$$8\sqrt{5} = 2 * y$$

$$y = \frac{8\sqrt{5}}{2}$$

$$y = 4\sqrt{5}$$

For x:

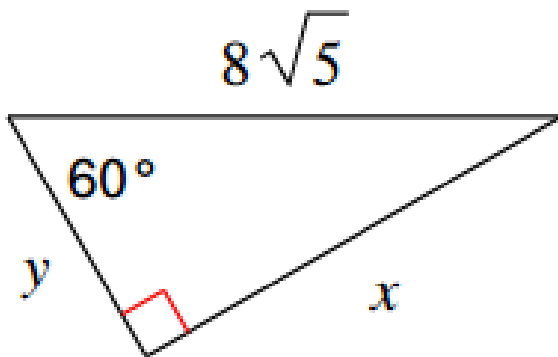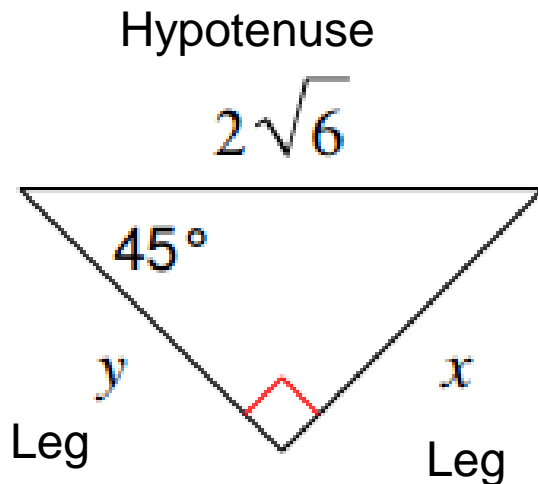$$x = \sqrt{3} * 4\sqrt{5}$$

$$x = 4\sqrt{3 * 5}$$

$$x = 4\sqrt{15}$$

EXAMPLES – BOTH TRIANGLES

Find the missing side lengths. Leave answers as simplified radicals (if necessary).

6.

Hypotenuse

$2\sqrt{6}$

45°

y

Leg

x

Leg

For x:

$$2\sqrt{6} = \sqrt{2} * x$$

$$x = \frac{2\sqrt{6}}{\sqrt{2}}$$

$$x = 2\sqrt{\frac{6}{2}}$$

$$\boldsymbol{x = 2\sqrt{3}}$$

For y:
Legs are $\cong$ in a $45° - 45° - 90°$ Triangle.

Thus, $\qquad \boldsymbol{y = 2\sqrt{3}}$