GEOMETRY UNIT

8-5: The Tangent Ratio

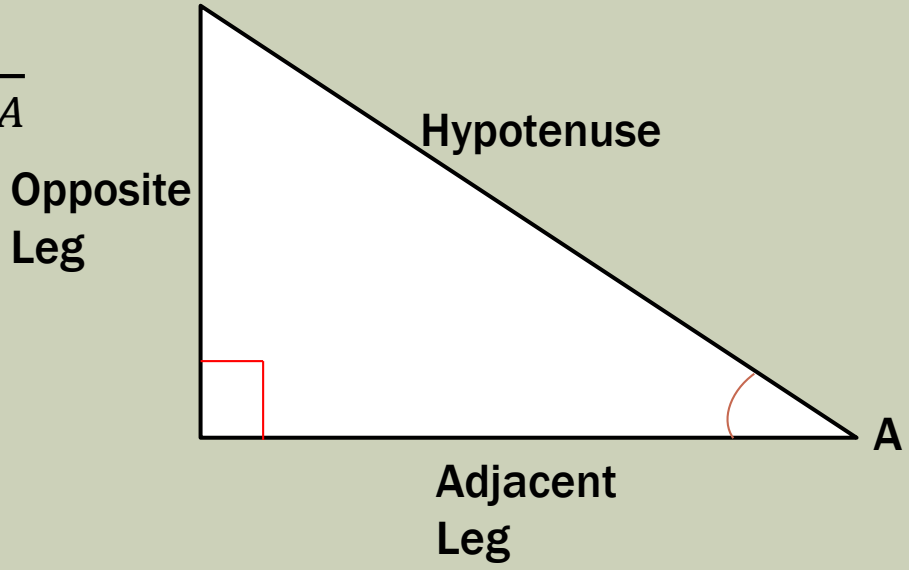
TRIGONOMETRY

- The word **Trigonometry** comes from the Greek words meaning "Triangle Measure."
- This material can be applied to any kind of triangle…
- But we will only be using this for right triangles.

TANGENT

- From the same triangle, only one acute angle ($< A$) is marked.
- The leg across from the angle is known as the **Opposite Leg** and the leg attached to the angle is known as the **Adjacent Leg**.
- The first of our **3 ratios** is known as the ***Tangent Ratio.***

$$\text{Tangent of} < A = \frac{leg\ opposite < A}{leg\ adjacent\ to < A}$$

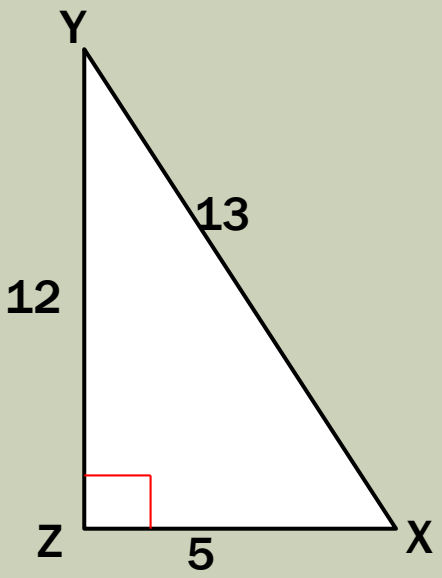


TANGENT EXAMPLES

- From the given triangle, find tan X and tan Y.

Tangent of $< X = \frac{leg\ opposite < X}{leg\ adjacent\ to < X}$

$$\frac{12}{5}$$



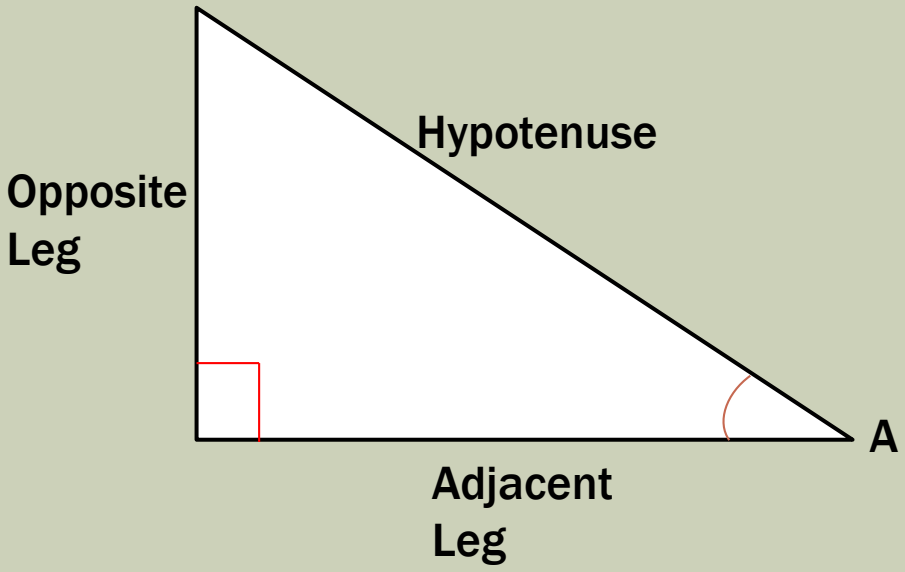
Tangent of $< Y = \frac{leg\ opposite < Y}{leg\ adjacent\ to < Y}$

$$\frac{5}{12}$$

SINE

- From the same triangle, only one acute angle ($< A$) is marked.
- Our next two ratios involve one of the legs, as well as the hypotenuse.
- This next ratio is known as the ***Sine Ratio.***

Sine of $< A = \frac{leg\ opposite < A}{hypotenuse}$



SINE EXAMPLES

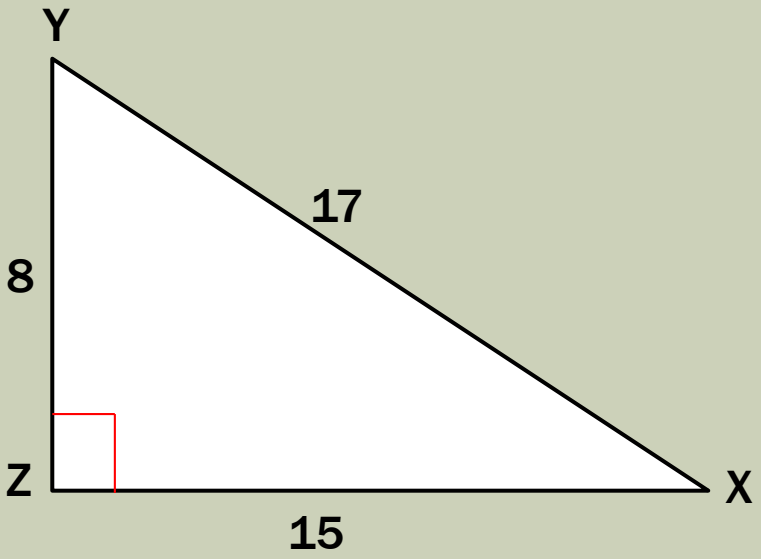
- From the given triangle, find sin X and sin Y.

$$\sin X = \frac{leg\ opposite < X}{hypotenuse}$$

$$\frac{8}{17}$$

$$\sin Y = \frac{leg\ opposite < Y}{hypotenuse}$$

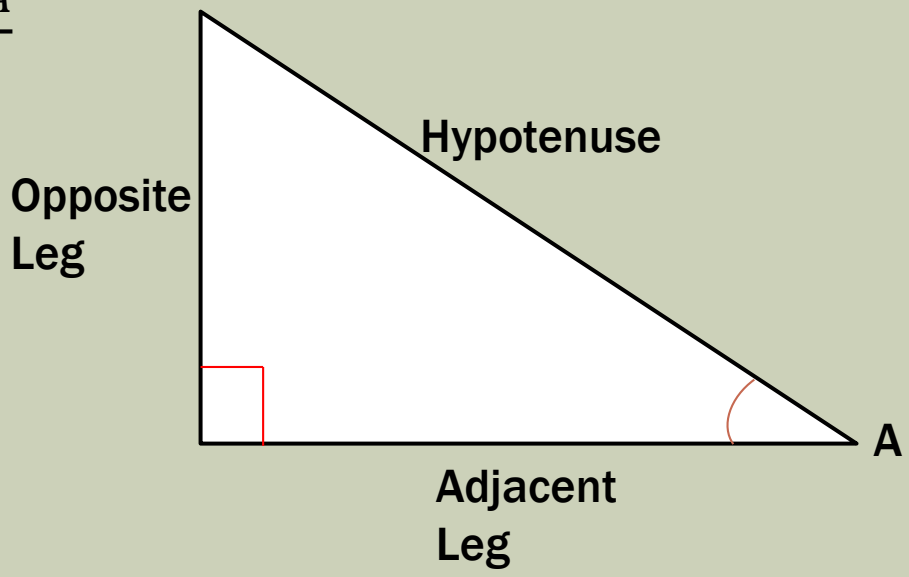
$$\frac{15}{17}$$



COSINE

- From the same triangle, only one acute angle ($< A$) is marked.
- The last ratio is known as the ***Cosine Ratio.***

Cosine of $< A = \frac{leg\ adjacent\ to < A}{hypotenuse}$

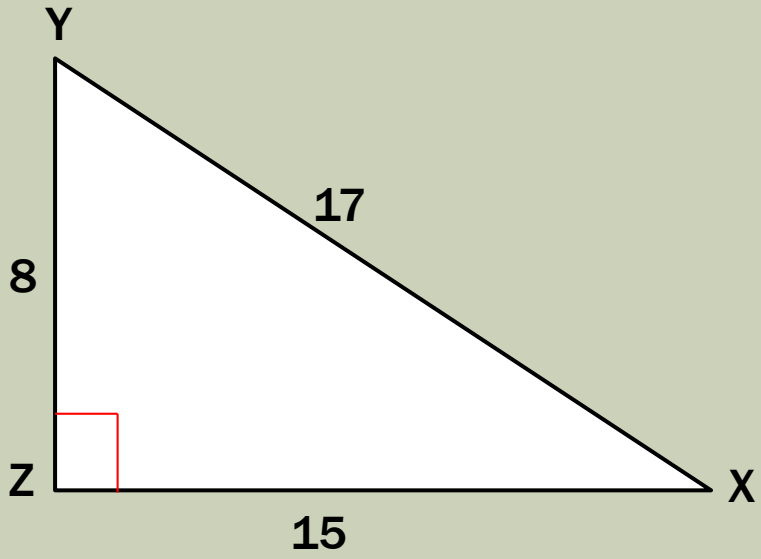


COSINE EXAMPLES

- From the given triangle, find cos X and cos Y.

$$\cos X = \frac{leg\ adjacent\ to < X}{hypotenuse}$$

$$\frac{15}{17}$$



$$\cos Y = \frac{leg\ adjacent\ to < Y}{hypotenuse}$$

$$\frac{8}{17}$$

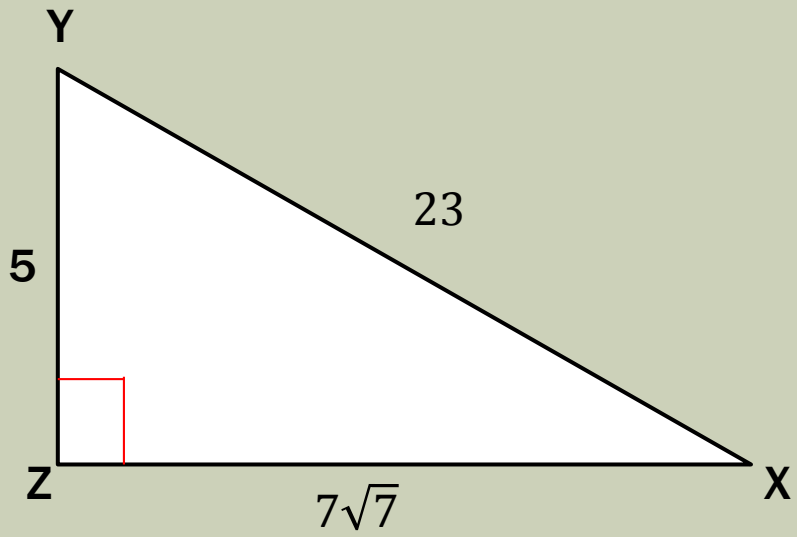
TRIG RATIOS

- **Content Objective:** Students will be able to solve for angles and sides of right triangles using the trig ratios of Sine, Cosine, and Tangent

- **Language Objective:** Students will be able to write trigonometric ratios using sides and angles of right triangles.

WARM-UP

- Find Sin X, Cos Y, Tan X, and Tan Y
- Solution:



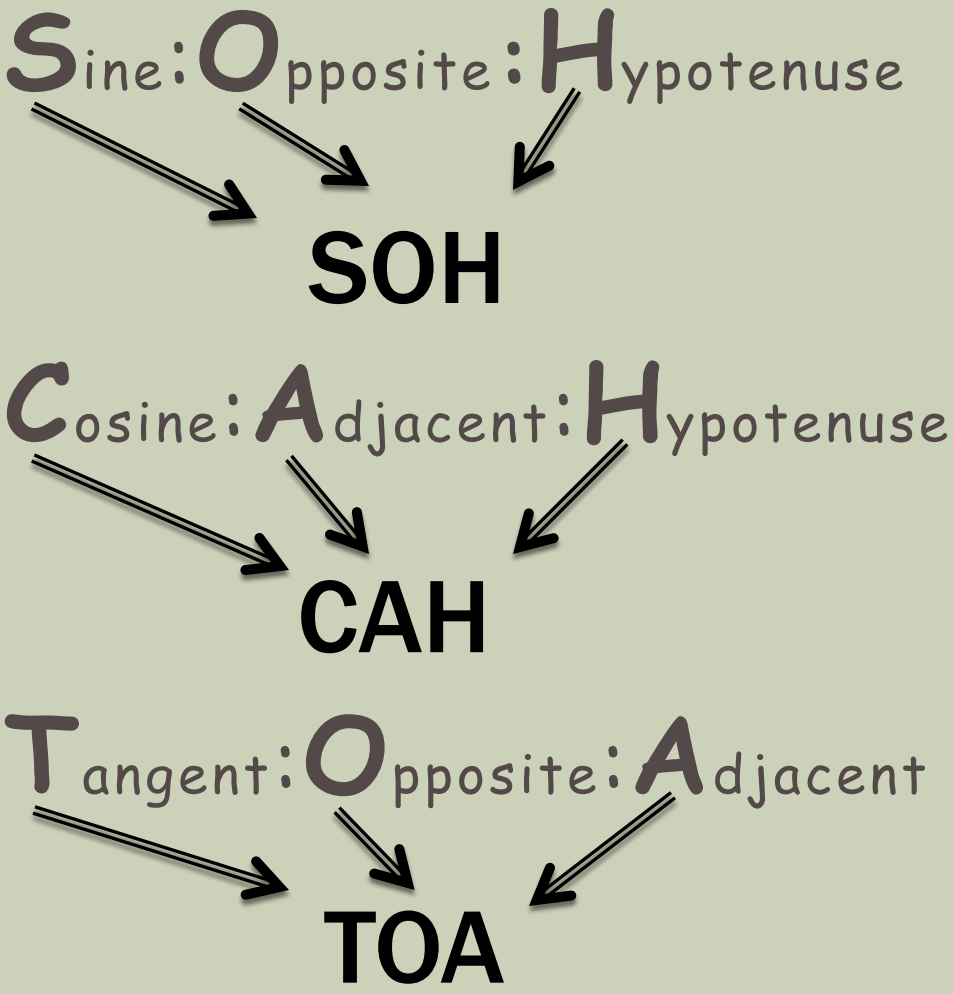
- $\sin X = \frac{5}{23}$
- $\cos Y = \frac{5}{23}$
- $\tan X = \frac{5}{7\sqrt{7}} = \frac{5\sqrt{7}}{49}$
- $\tan Y = \frac{7\sqrt{7}}{5}$

PUTTING IT ALL TOGETHER

- In trigonometry, there is a saying that helps with memorizing how to set up the ratios of Sine, Cosine and Tangent.

- See if you can get it from this:

PUTTING IT ALL TOGETHER



PUTTING IT ALL TOGETHER

- All together, we have…

SOH-CAH-TOA

TRIG WITH ANGLES

- Trig Ratios can also be used to find the values of specific angles.
- For example, you can write $\tan 10°$ to represent the tangent of any angle of degree measure 10.
- You can find these values by using either a calculator, or a table of values.
- (i.e. there is a table of trig values on page 311 of your textbook).

Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000				

TRIG WITH ANGLES

- Examples:

1.) $\tan 10° \approx \mathbf{0.1763}$

2.) $\sin 25° \approx \mathbf{0.4226}$

3.) $\cos 44° \approx \mathbf{0.7193}$

4.) $\tan 60° \approx \mathbf{1.7321}$

5.) $\sin 45° \approx \mathbf{0.7071}$

6.) $\cos 30° \approx \mathbf{0.8660}$

TRIG WITH ANGLES

- Using the trig values of specific angles is helpful for finding missing sides of a triangle.
- Example: Find the value of x.

Solution:

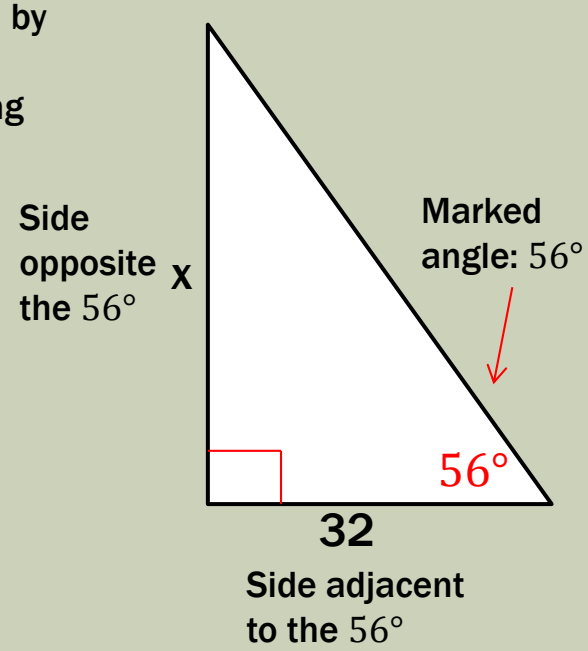
$$\tan 56° = \frac{x}{32}$$

$$x = 32 * \tan 56°$$

$$x = 32 * 1.4826$$

$y = 47.4432$ **or** $y \approx 47.4$

You get this decimal by either checking the table, or just plugging **tan 56°** into your calculator.



TRIG WITH ANGLES

- Now you try
- Example: Find the values of x and y.

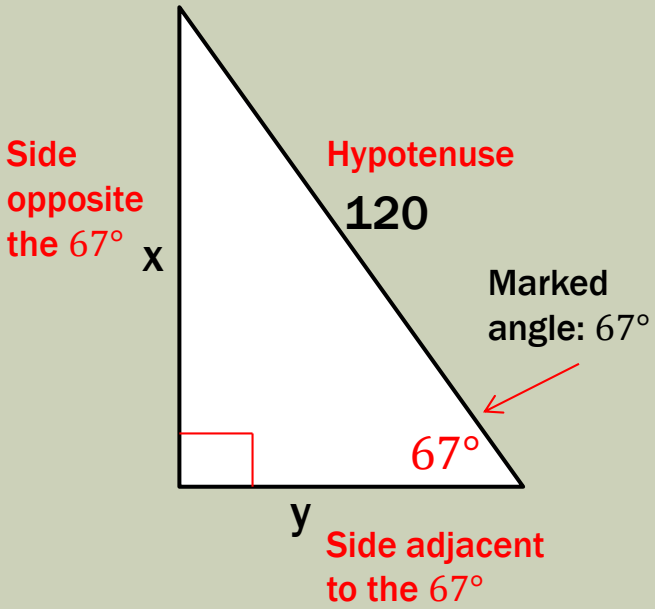
Solution (For x):

$$\sin 67° = \frac{x}{120}$$

$$x = 120 * \sin 67°$$

$$x = 120 * 0.9205$$

$x = 110.46$ **or** $x \approx 110$



TRIG WITH ANGLES

- Now you try
- Example: Find the values of x and y.

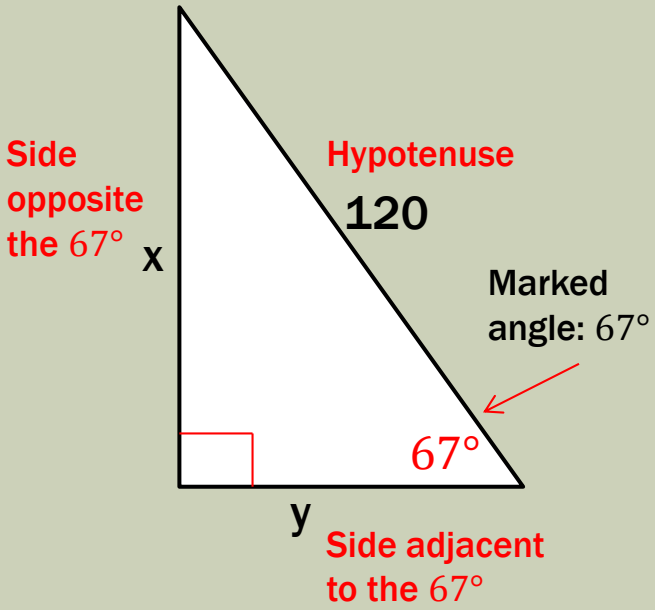
Solution (For y):

$$\cos 67° = \frac{y}{120}$$

$$y = 120 * \cos 67°$$

$$y = 120 * 0.3907$$

$y = 46.884$ **or** $y \approx 47$



WHAT IF I DON'T GIVE YOU THE ANGLE?

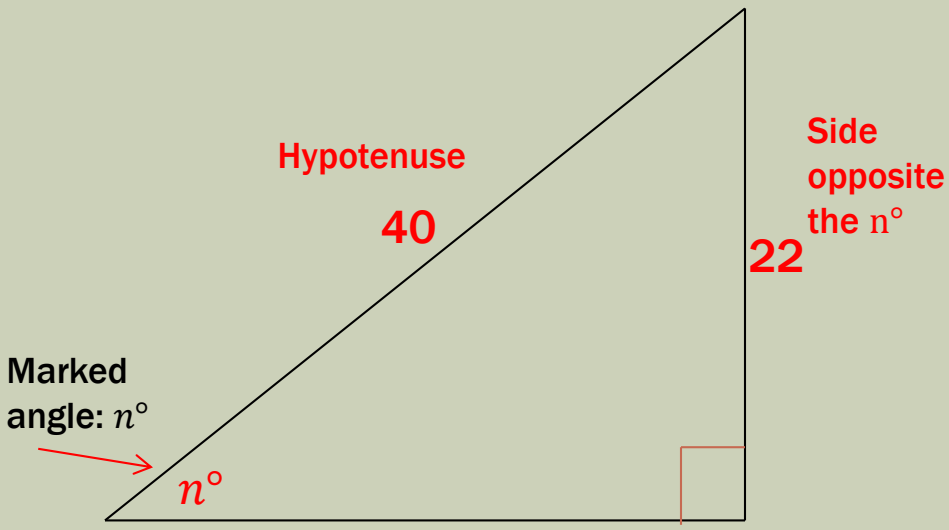
- Find the measure of n to the nearest integer.

Solution:

$$\sin n^\circ = \frac{22}{40}$$

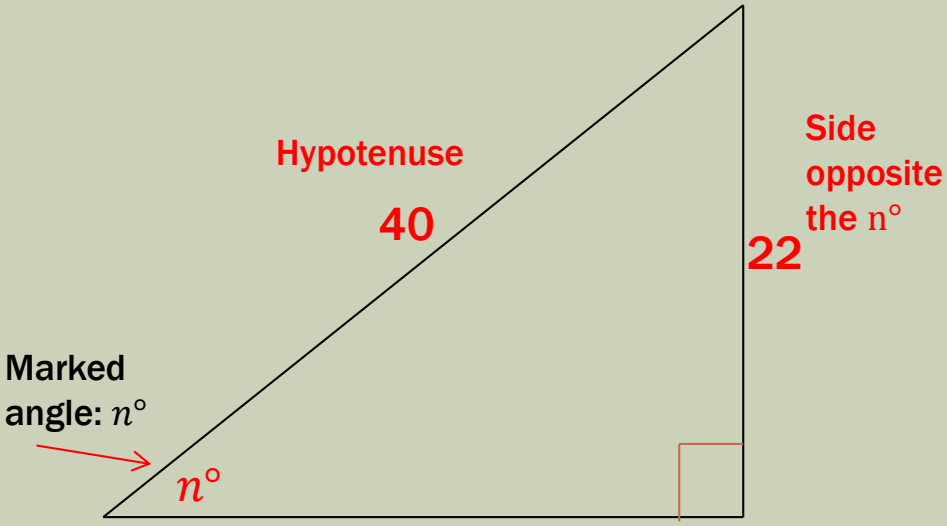
$$\sin n^\circ = 0.5500$$

From here, you have a choice:
Either look for 0.5500 (or the closest value to it) on the table...
Or let your calculator do it the following way:
1.) Go to the button that reads "2nd"
2.) Hit the "sin" button. If it went well, then "$\sin^{-1}($ " should appear on the screen.
3.) Put the decimal value (0.5500) in the and press "enter"
4.) Round your answer to the nearest integer, and there you go.



WHAT IF I DON'T GIVE YOU THE ANGLE?

- If all went well, you should have
- $\sin^{-1}(0.5500) = 33.3670 \approx \mathbf{33}$



FINDING ANGLES

- This same process can be applied when solving for angles using Cosine and Tangent.
- Give it a try with these examples: Find the value of x.
- **1.)** $\cos x° = 0.6678$

$\mathbf{\cos^{-1}(0.6678) = 48.1025 \approx 48}$

- **2.)** $\tan x° = 0.3246$

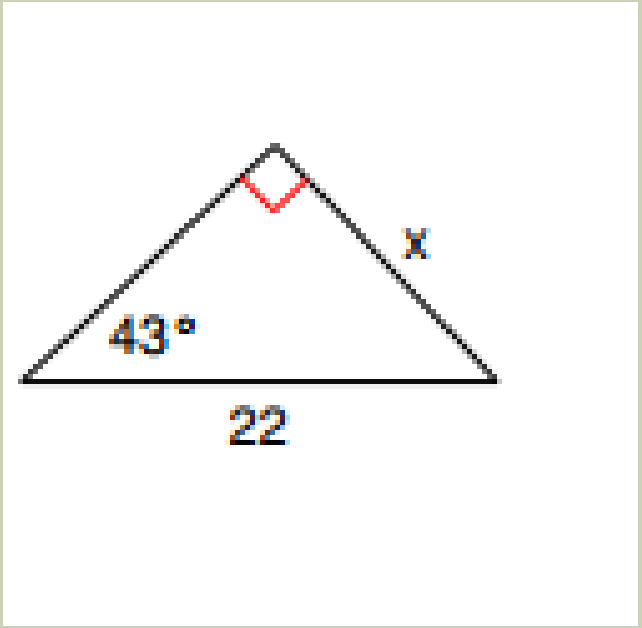
$\mathbf{\tan^{-1}(0.3246) = 17.9834 \approx 18}$

Note: When you have to divide to get the decimal, it is best to round to 4 decimal places.

FINAL CHECK

- Solve the value of x using trig ratios.

1.)



Solution:

$$\sin 43° = \frac{x}{22}$$

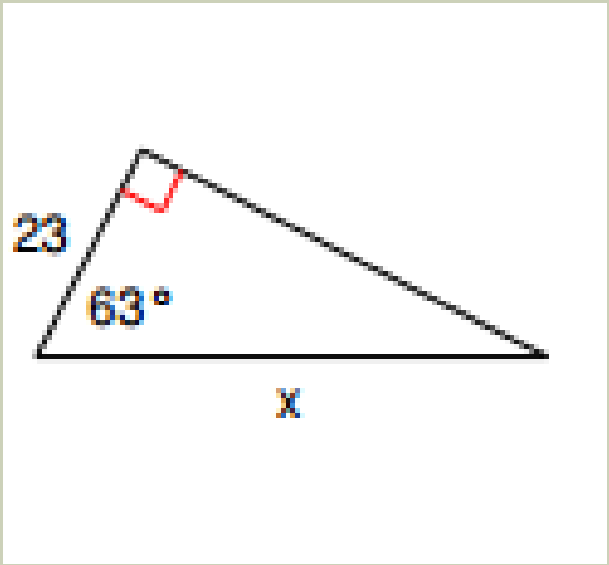
$$x = 22 \times \sin 43°$$

$$x \approx 15.004$$

FINAL CHECK

- Solve the value of x using trig ratios.

2.)



Solution:

$$\cos 63° = \frac{23}{x}$$

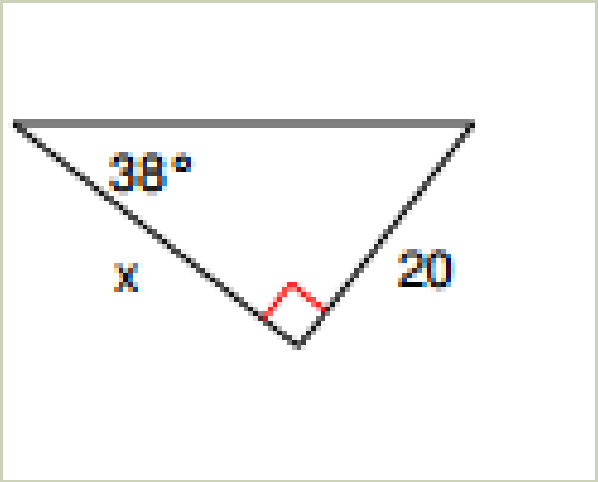
$$x = \frac{23}{\cos 63°}$$

$$x \approx 50.6619$$

FINAL CHECK

- Solve the value of x using trig ratios.

3.)



Solution:

$$\tan 38° = \frac{20}{x}$$

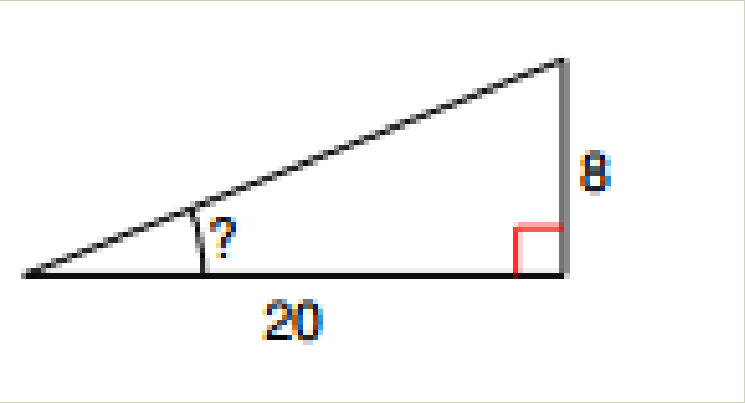
$$x = \frac{20}{\tan 38°}$$

$$x \approx 25.5988$$

FINAL CHECK

- Solve the value of x using trig ratios.

4.)



*Just pretend "?" is x.

Solution:

$$\tan x° = \frac{8}{20}$$

$$\tan x° = 0.4$$

$$x = \tan^{-1}(0.4) = 21.8014 \approx \mathbf{22}$$