GEOMETRY UNIT 9

9-1: Circle Basics

And

9-2: Tangent Properties

CIRCLES

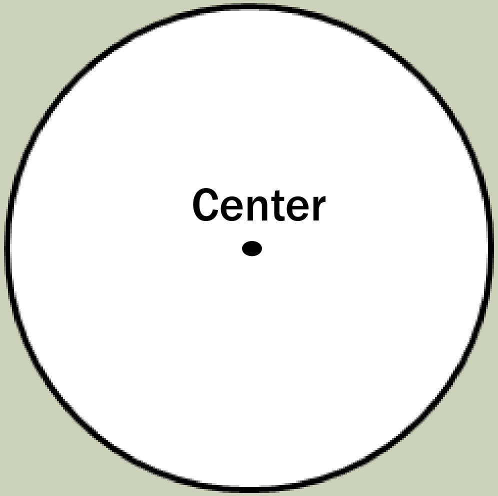
- Content Objective: Students will be able to solve for missing lengths in circles.

- Language Objective: Students will be able to identify the types of lines in circles through their notation and their properties.

CIRCLES – BASIC TERMS

- **Circle**: A set of points in a plane that are all equidistant (the same given distance) from a given point in that plane.

- **Center**: The point in the plane that all points of the circle are equidistant to.



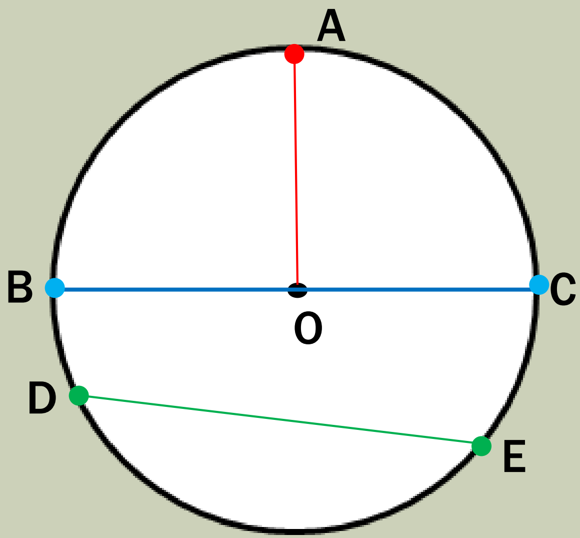
LINES INSIDE THE CIRCLE

- **Radius**: The line that represents the distance from any given point on the circle to the center.
- **Chord**: A segment whose endpoints lie on a circle.
- **Diameter**: A chord that goes through the center of the circle.

Radius: $\overline{OA}$

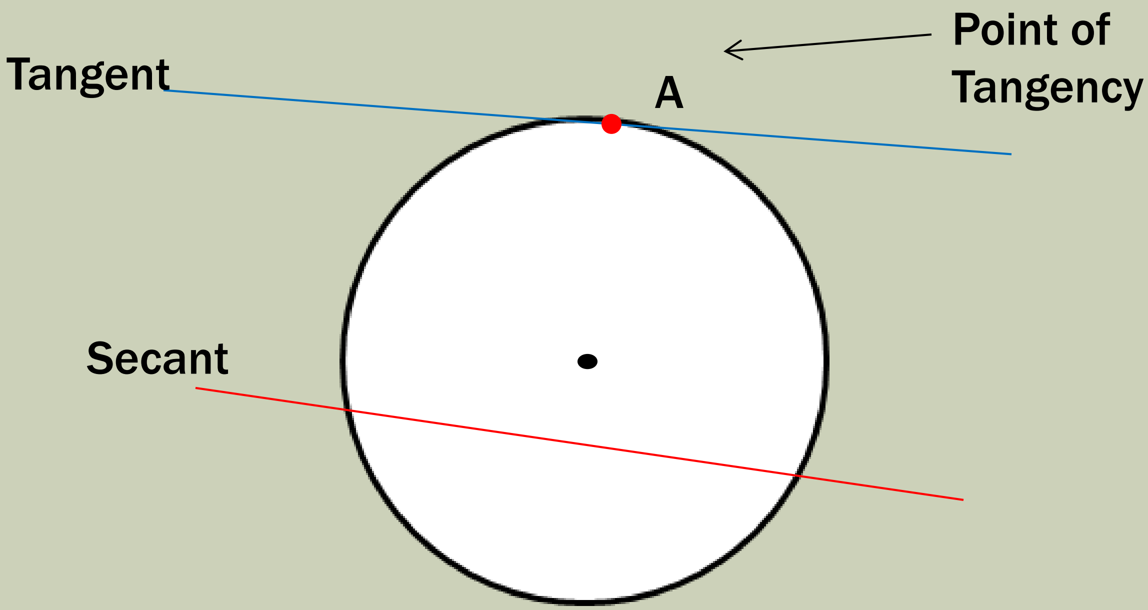
Chord: $\overline{DE}$

Diameter: $\overline{BC}$



LINES THROUGH THE CIRCLE

- **Secant**: A line that goes through a circle, crossing at two points.
- **Tangent**: A line in the plane of a circle that intersects the circle in exactly on point, called the point of tangency.



LINES PRACTICE

- Identify the type of line based off the picture and the notation given.

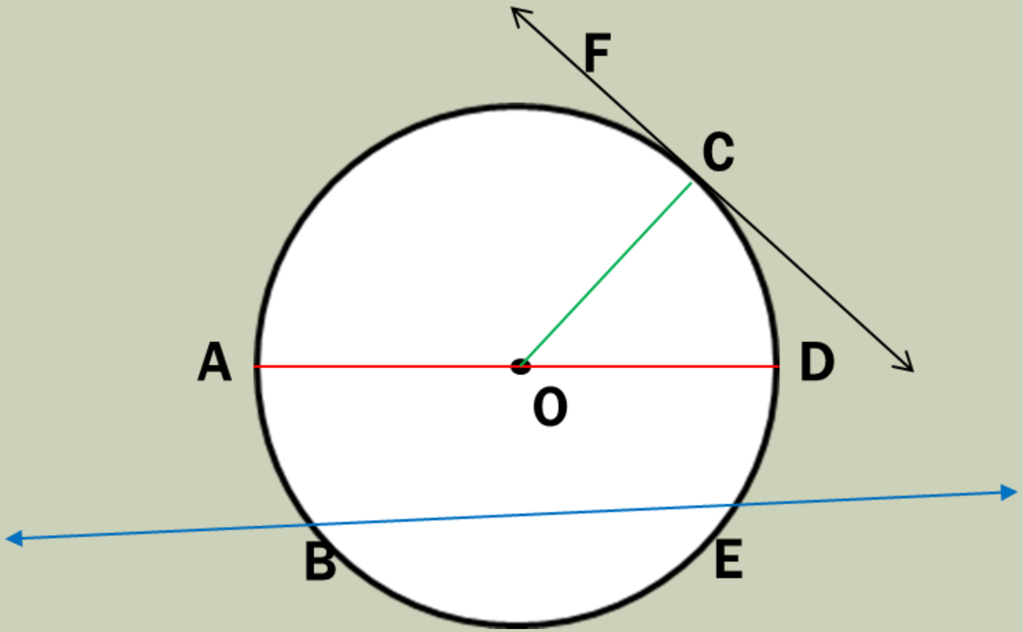
1.) $\overline{AD}$ Diameter

2.) $\overline{BE}$ Chord

3.) $\overleftrightarrow{BE}$ Secant

4.) $\overleftrightarrow{FC}$ Tangent

5.) $\overline{OC}$ Radius

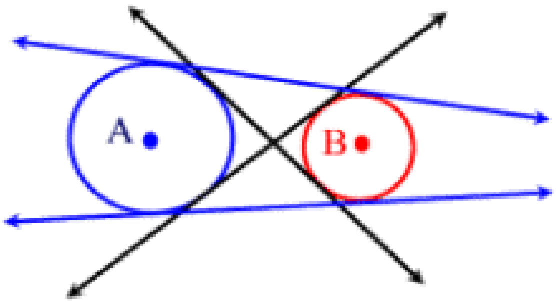


COMMON TANGENTS

- A Line that is tangent to each of two coplanar circles is called a **common tangent**.
- Between two coplanar circles, there can be no common tangents, or as many as 4…
- It all depends on how the circles are placed.
- Examine the chart on the following slide to gain an understanding.

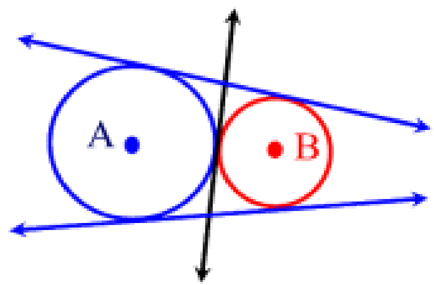
Common Tangents – Chart

4 Common Tangents
(2 completely separate circles)



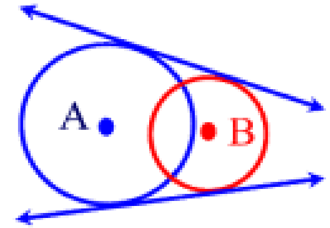
2 external tangents (blue)
2 internal tangents (black)

3 Common Tangents
(2 externally tangent circles)



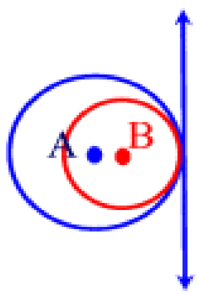
2 external tangents (blue)
1 internal tangent (black)

2 Common Tangents
(2 overlapping circles)



2 external tangents (blue)
0 internal tangents

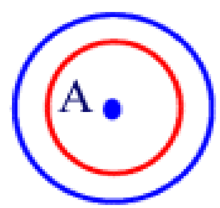
1 Common Tangent
(2 internally tangent circles)



1 external tangent (blue)
0 internal tangents

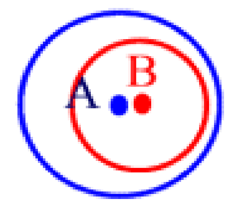
0 Common Tangents

(2 concentric circles)
Concentric circles are circles with the same center.



0 external tangents
0 internal tangents

(one circle floating inside the other, without touching)



0 external tangents
0 internal tangents

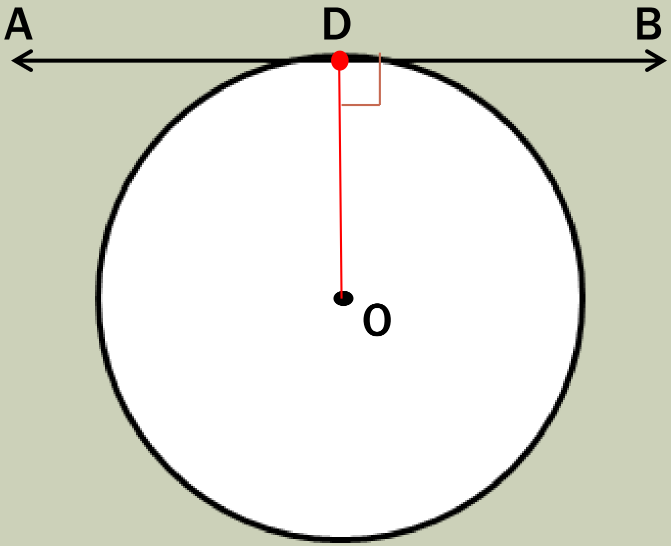
THEOREMS WITH TANGENTS

- **Theorem 9-1**: If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

If: $\overleftrightarrow{AB}$ is a tangent

D is the point of tangency

Then: $\overleftrightarrow{AB} \perp \overline{OD}$

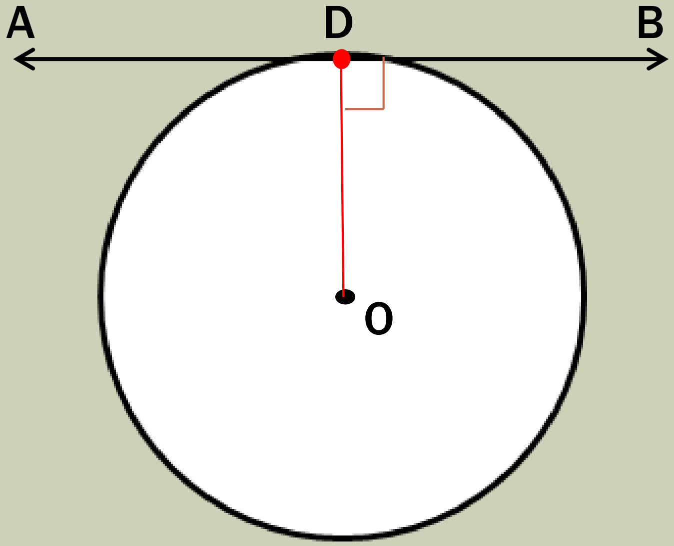


THEOREMS WITH TANGENTS

- **Theorem 9-2**: If a line in the plane of a circle is perpendicular to a radius at its other endpoint, then the line is tangent to the circle.

If: $\overleftrightarrow{AB} \perp$ radius $\overline{OD}$ at point D.

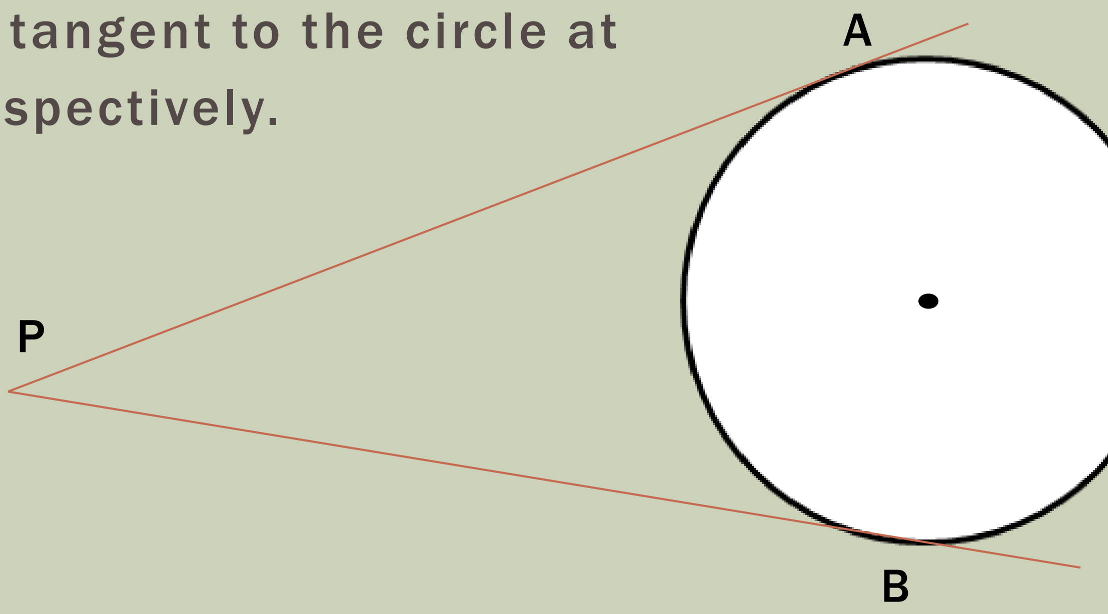
Then: $\overleftrightarrow{AB}$ is tangent to the circle



COROLLARY WITH TANGENTS

- **Corollary**: Tangents to a circle from a point are congruent

If: $\overline{PA}$ and $\overline{PB}$ are tangent to the circle at points A and B, respectively.



Then: $\overline{PA} \cong \overline{PB}$

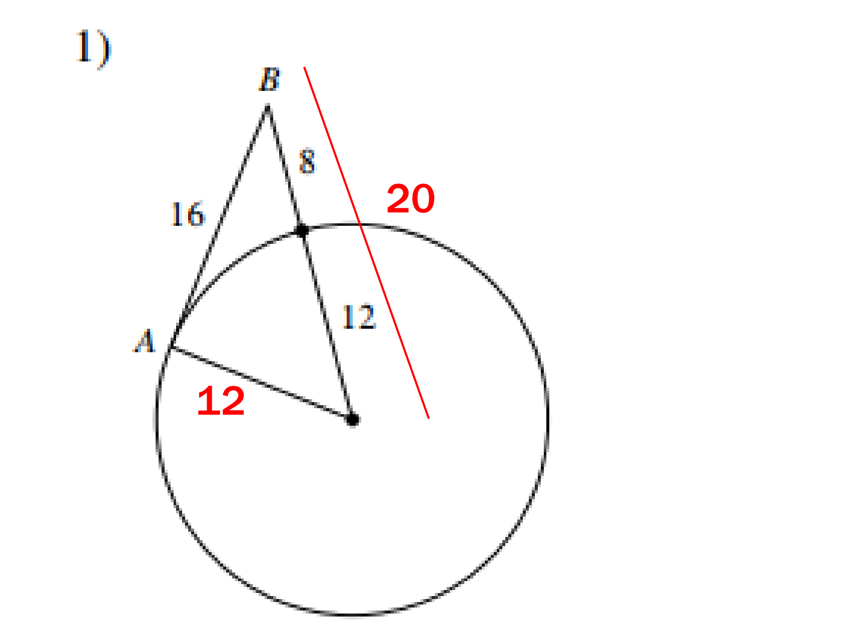
WHERE WILL YOU SEE THESE THEOREMS USED

- Since these theorems tell us about properties related to tangent, we can use them to confirm the existence of tangent lines…

- Or find the measures of lines when given tangents.

IDENTIFYING TANGENT LINES

- Determine if the line AB is tangent to the circle



From the theorems, we can determine that we should have a right triangle in order for $\overline{AB}$ to be a Tangent.

So we must check if this is a right triangle…

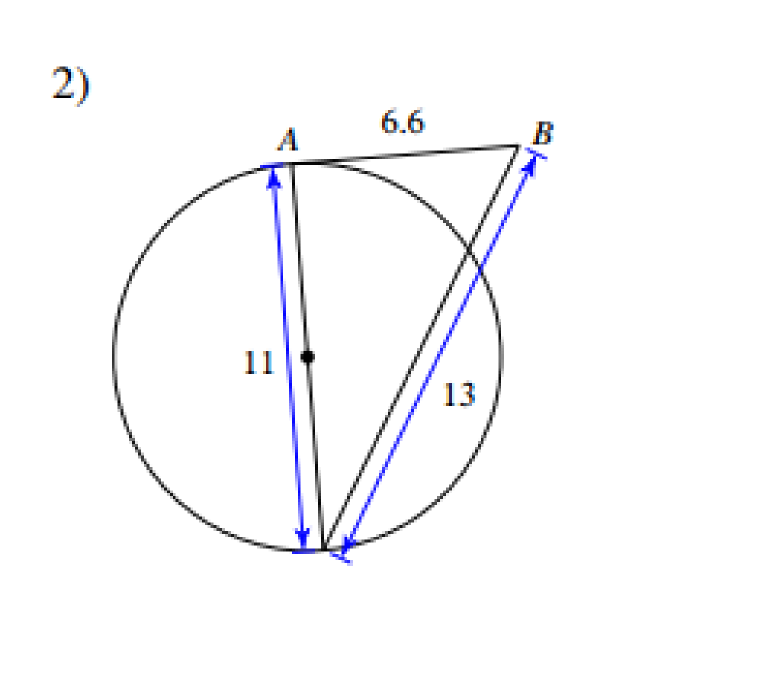
Which we can do by using the Pythagorean Theorem

You should discover that this is a Pythagorean Triple (12-16-20), so this is a right triangle.

Thus, $\overline{AB}$ is a tangent line.

IDENTIFYING TANGENT LINES

- Determine if the line AB is tangent to the circle



Just like the previous problem, you must check to see if this a right triangle using the Pythagorean Theorem.

You should have

$$11^2 + 6.6^2 = 13^2$$

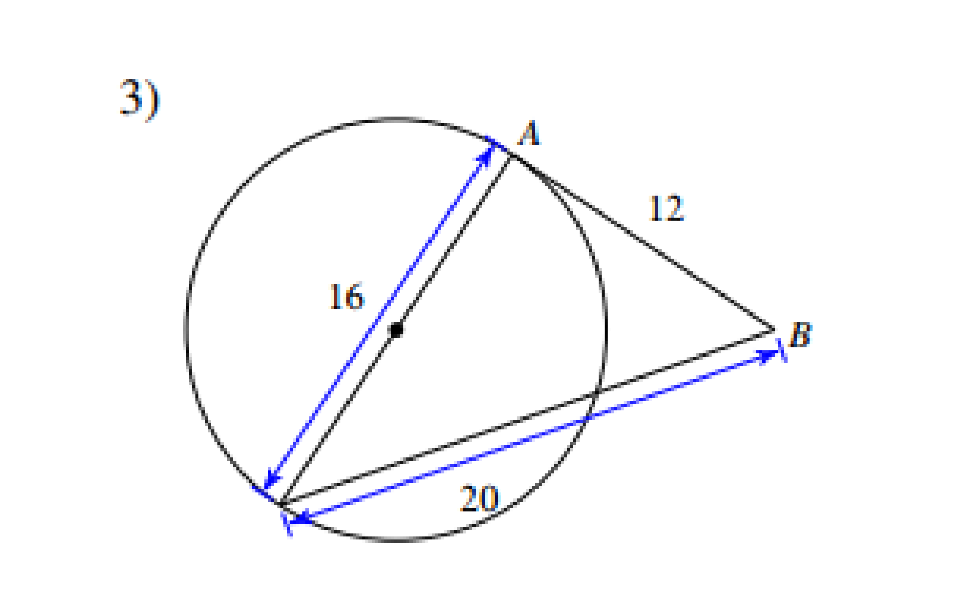
$$121 + 43.56 = 169$$

$$\mathbf{164.56 \neq 169}$$

Thus, $\overline{AB}$ is NOT a tangent line

IDENTIFYING TANGENT LINES – TRY THESE IN YOUR GROUP

- Determine if the line AB is tangent to the circle

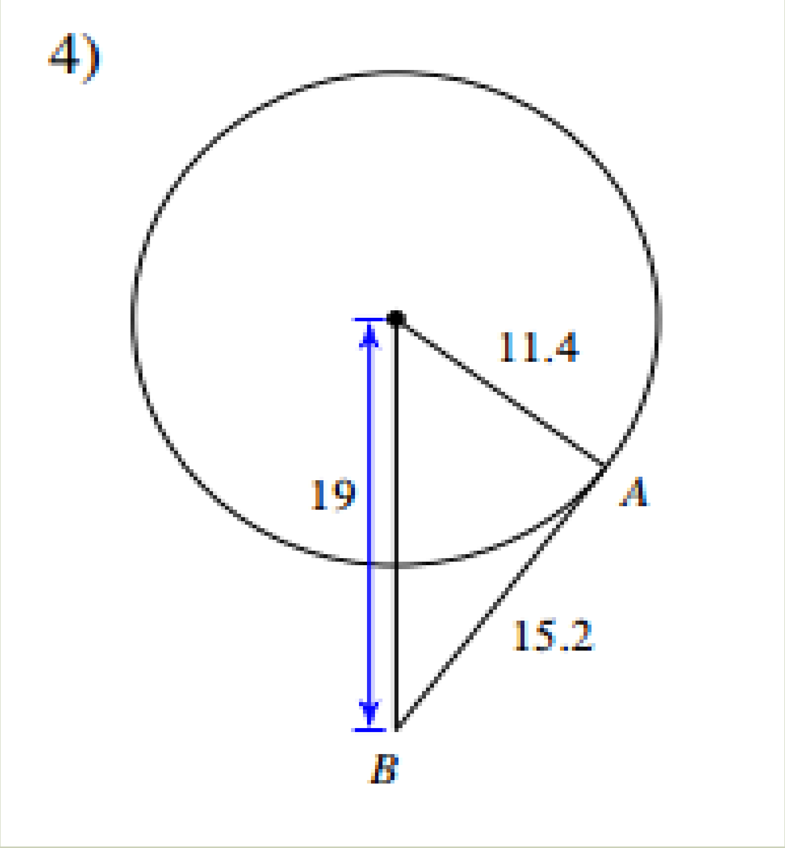


You should have another Pythagorean triple

Thus, $\overline{AB}$ is a tangent line

IDENTIFYING TANGENT LINES

- Determine if the line AB is tangent to the circle



You will need the Pythagorean Theorem.

You should have

$$11.4^2 + 15.2^2 = 19^2$$

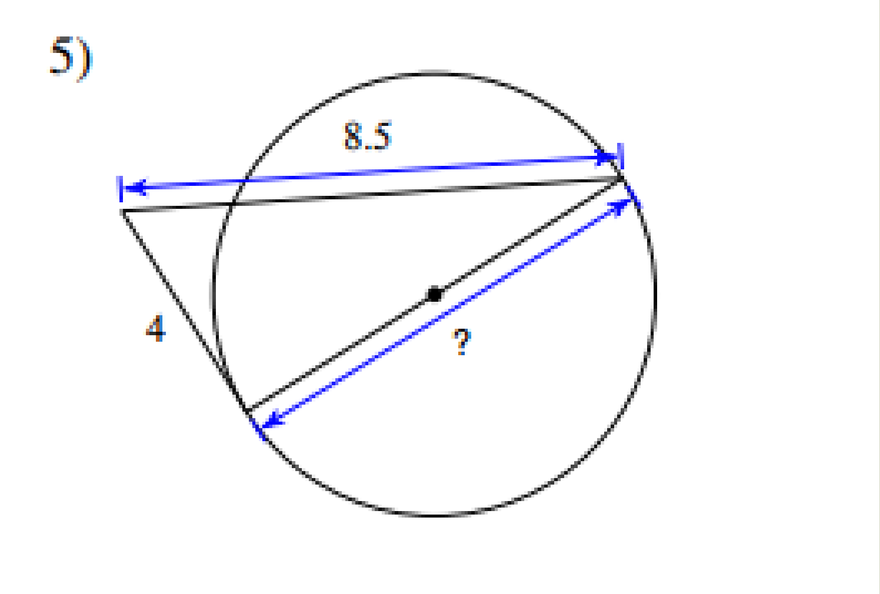
$$129.96 + 231.04 = 361$$

$$\mathbf{361 = 361}$$

Thus, $\overline{AB}$ is a tangent line

USING TANGENTS TO FIND LENGTHS

- Find the segment length indicated (pretend the question mark is an "x"). You may use rounded decimals in your answers.



Since we have a tangent line, then we have a right triangle. Thus, we can use the Pythagorean theorem to find the missing side:

$$x^2 + 4^2 = 8.5^2$$

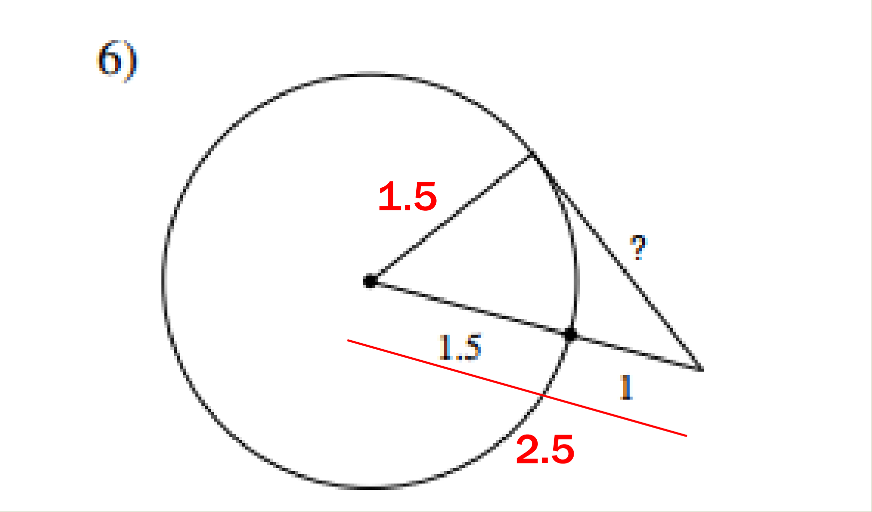
$$x^2 + 16 = 72.25$$

$$x^2 = 56.25$$

$$x = \sqrt{56.25} = \mathbf{7.5}$$

USING TANGENTS TO FIND LENGTHS – TRY THE REST IN YOUR GROUPS

- Find the segment length indicated (pretend the question mark is an "x"). You may use rounded decimals in your answers.



First get all the sides in order, then use the Pythagorean Theorem:

$$x^2 + 1.5^2 = 2.5^2$$

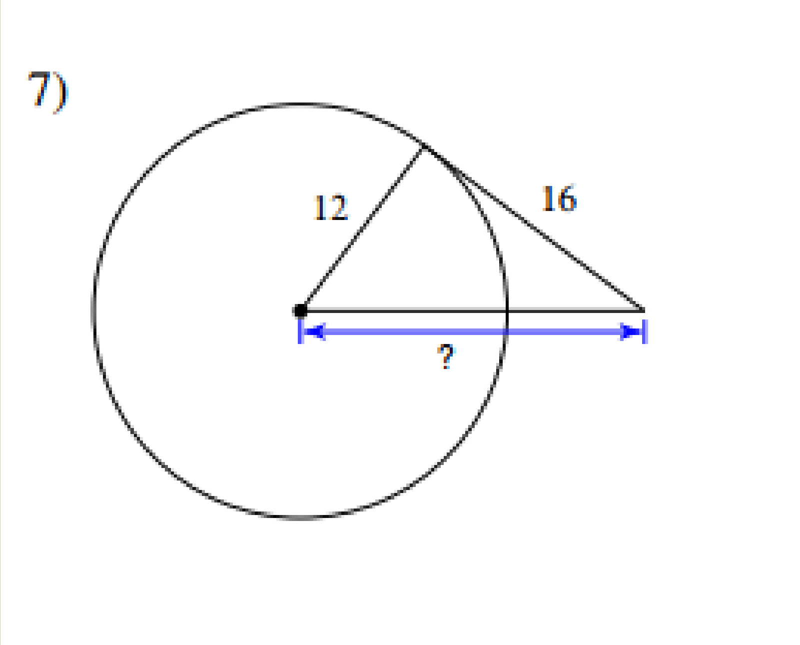
$$x^2 + 2.25 = 6.25$$

$$x^2 = 4$$

$$x = \sqrt{4} = \mathbf{2}$$

USING TANGENTS TO FIND LENGTHS

- Find the segment length indicated (pretend the question mark is an "x"). You may use rounded decimals in your answers.



You have seen this a few times already:

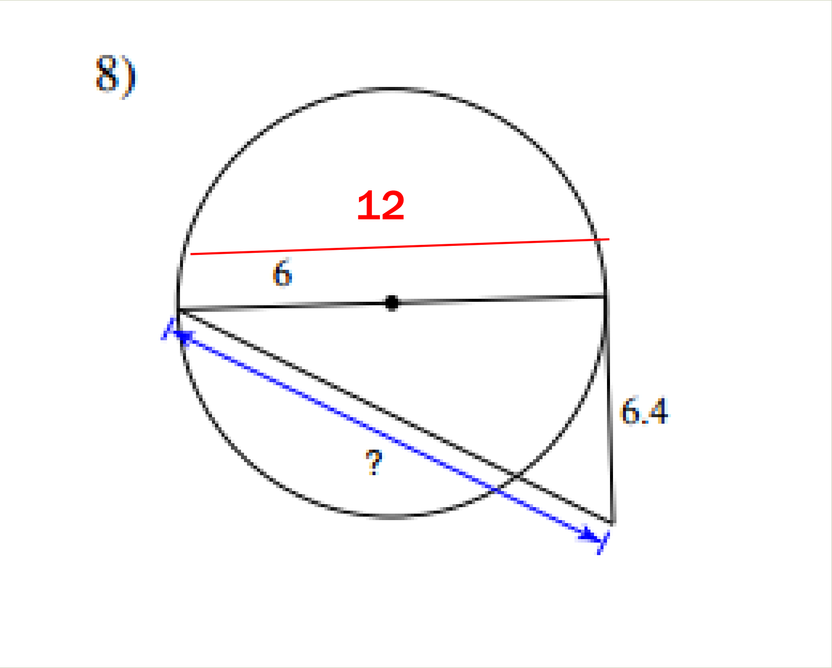
This a Pythagorean Triple, with 12 and 16 as the legs.

Thus,

$$x = 20$$

USING TANGENTS TO FIND LENGTHS

- Find the segment length indicated (pretend the question mark is an "x"). You may use rounded decimals in your answers.



Use the Pythagorean Theorem:

$$12^2 + 6.4^2 = x^2$$

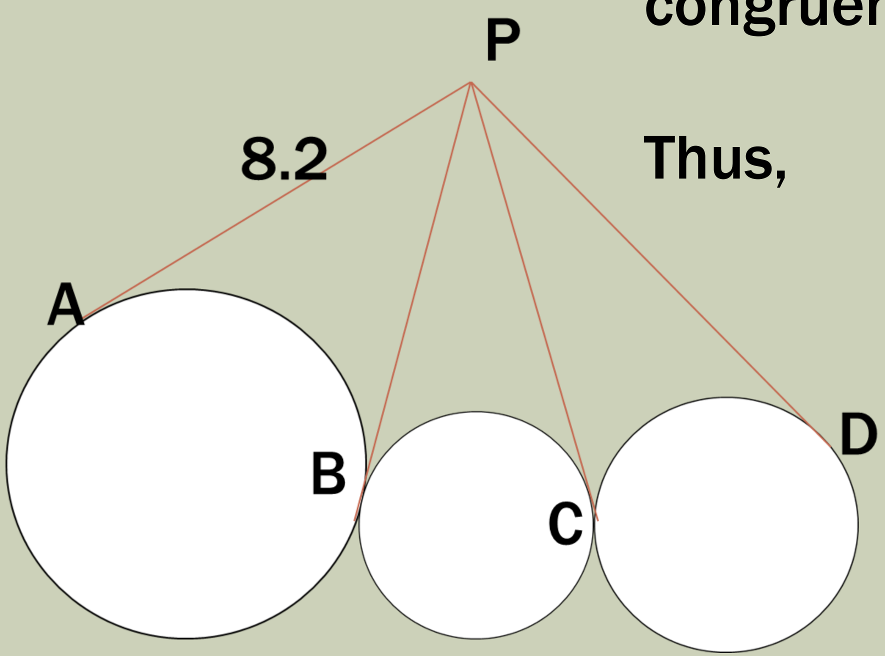
$$144 + 40.96 = x^2$$

$$184.96 = x^2$$

$$x = \sqrt{184.96} = \mathbf{13.6}$$

WARM-UP: TANGENTS PRACTICE

- The diagram below shows tangent lines and circles. Find PD.



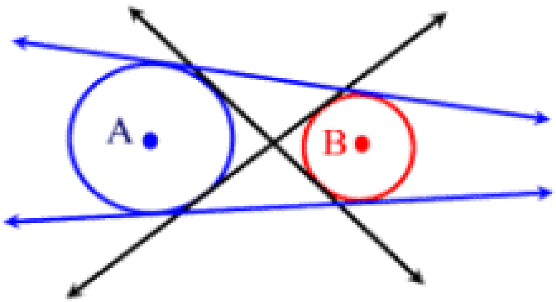
Using the corollary, since all the lines are coming from the point P, then they are all congruent to each other.

Thus,

$$PD = 8.2$$

Common Tangents – Chart

4 Common Tangents
(2 completely separate circles)



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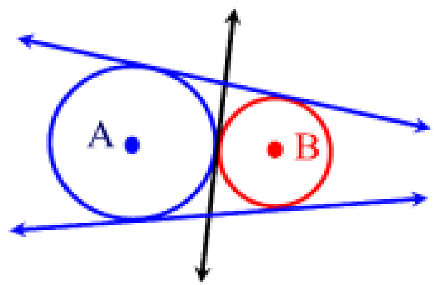

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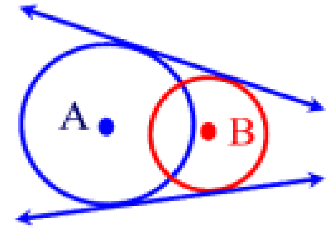

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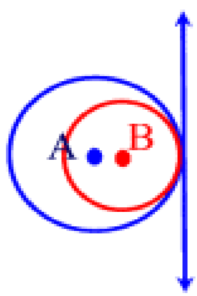

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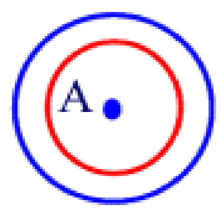

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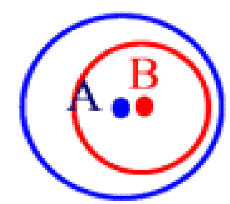
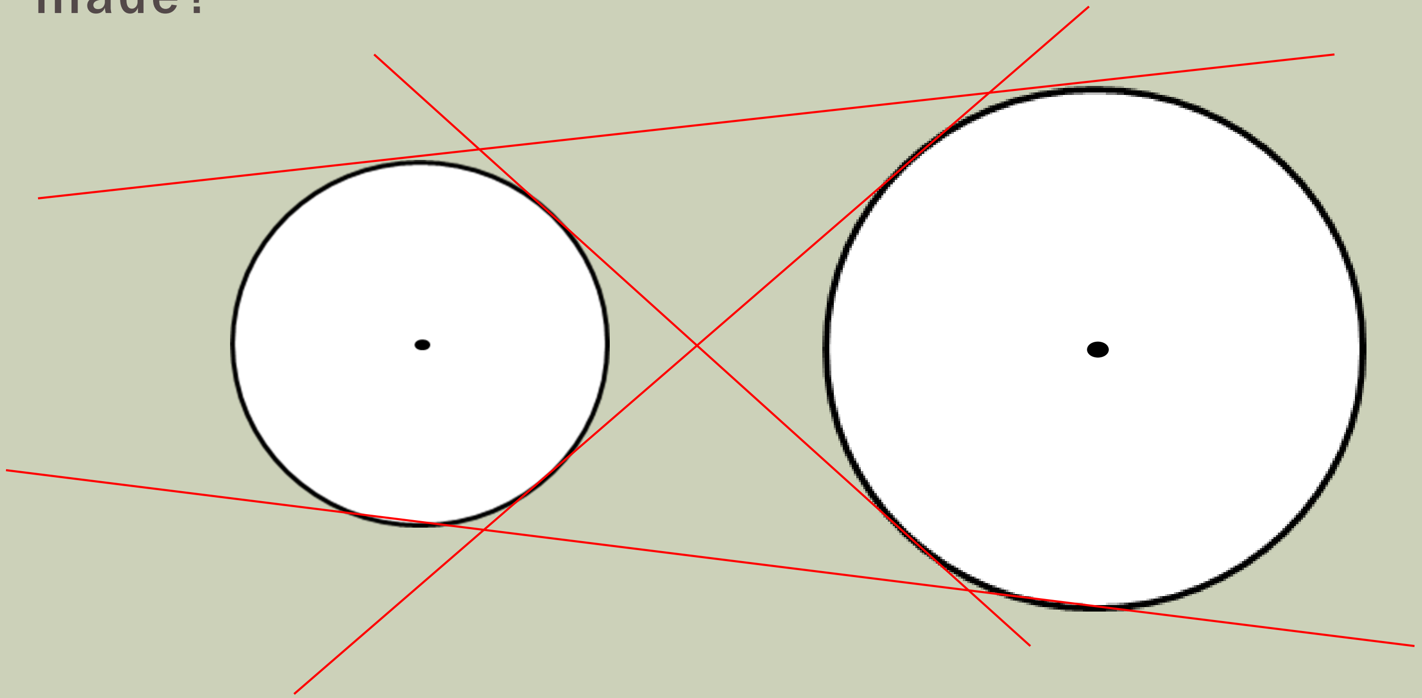
(one circle floating inside the other, without touching)



0 external tangents
0 internal tangents

COMMON TANGENTS PRACTICE

- From each of the given circles, how many common tangents be made?



4 Common Tangents

COMMON TANGENTS PRACTICE

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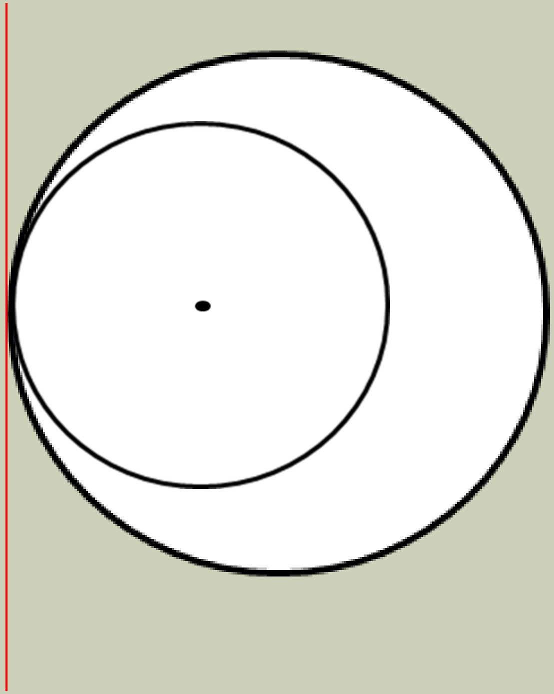

1 Common Tangent

COMMON TANGENTS PRACTICE

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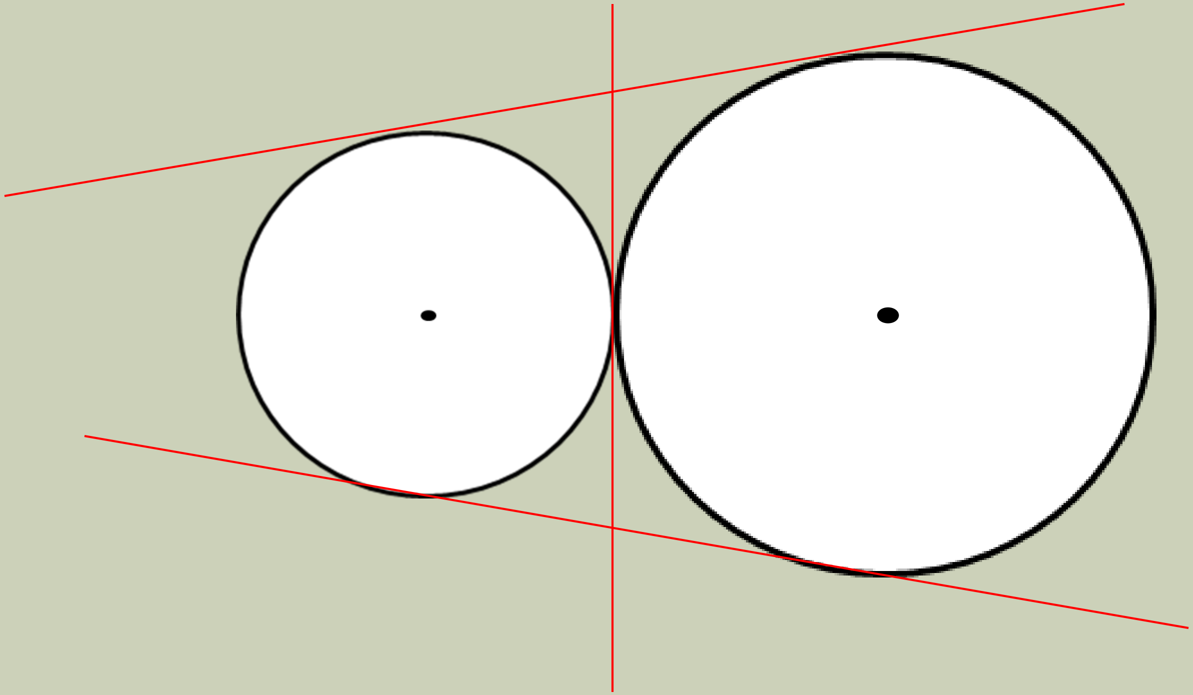
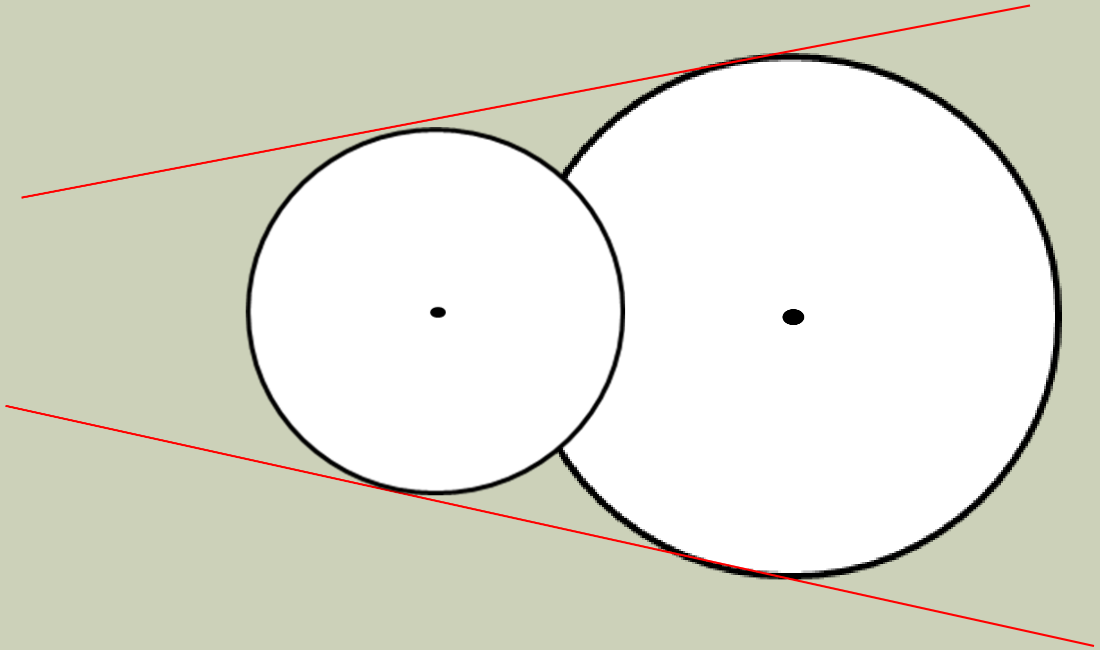


3 Common Tangents

COMMON TANGENTS PRACTICE

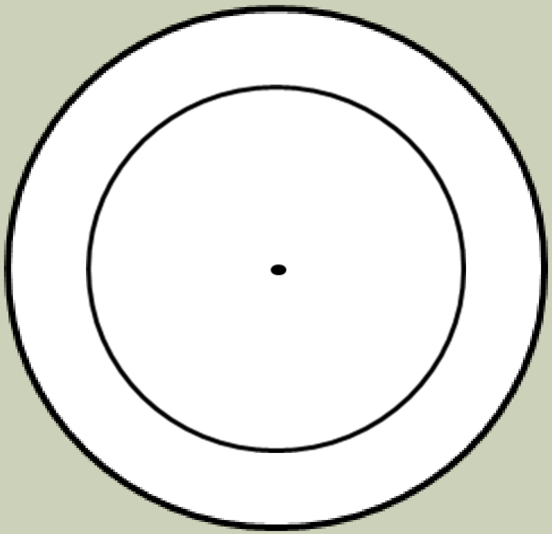
- From each of the given circles, how many common tangents be made?



2 Common Tangents

COMMON TANGENTS PRACTICE

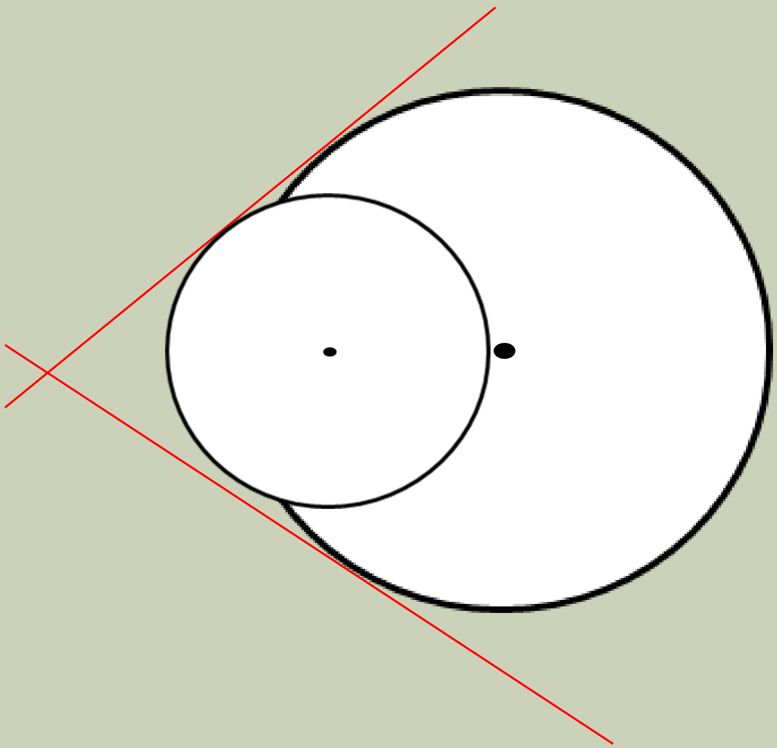
- From each of the given circles, how many common tangents be made?



No Common Tangents

COMMON TANGENTS PRACTICE

- From each of the given circles, how many common tangents be made?



2 Common Tangents