GEOMETRY UNIT 9

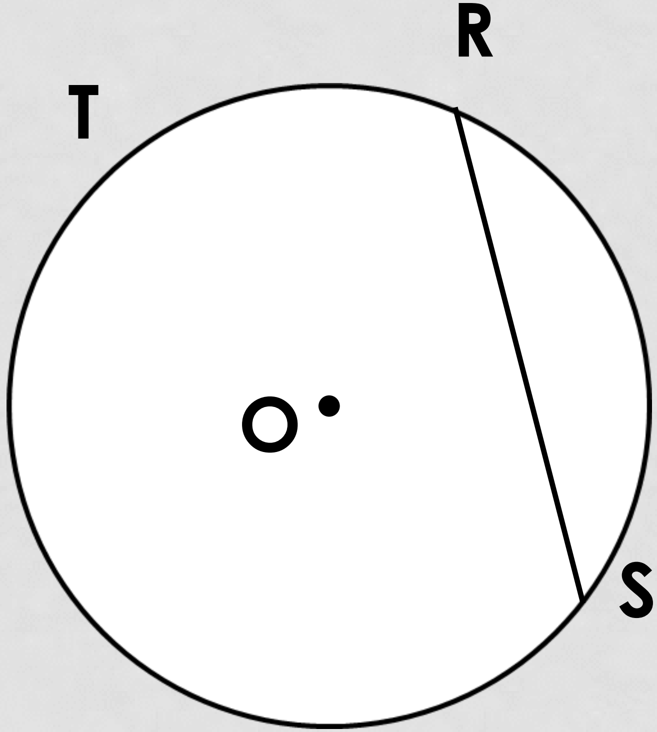
9-4: ARCS AND CHORDS

ARCS AND CHORDS

- **Content Objective**: Students will be able to find the measures of arcs and chords in circles.
- **Language Objective**: Students will be able to identify properties of arcs and chords from theorems and examples.

ARCS AND CHORDS

- In $\odot O$, $\overline{RS}$ cuts off two arcs, minor arc $\widehat{RS}$, and major arc $\widehat{RTS}$.
- From the two arcs, we say that the minor arc, $\widehat{RS}$, is called *the arc of* $\overline{RS}$.



The arc of $\overline{RS}$ is $\widehat{RS}$.

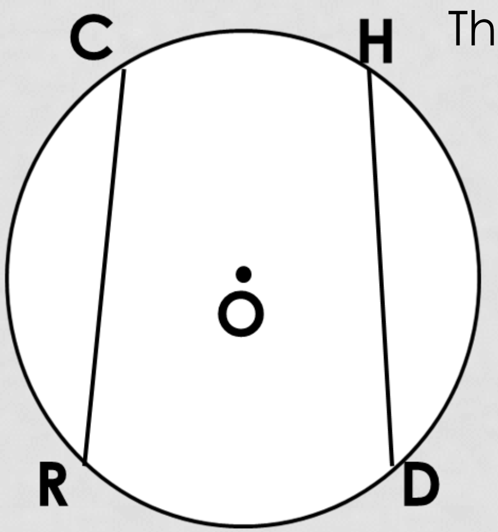
ARCS AND CHORDS

Theorem 9-4: In the same circle, or in congruent circles:

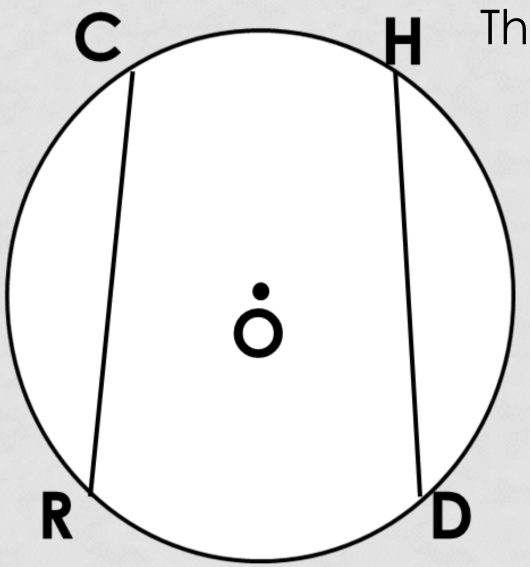
(1) Congruent arcs have congruent chords;

(2) Congruent chords have congruent arcs.

For $\odot O$, we have the two conditions:



(1) If: $m\,\widehat{CR} \cong m\,\widehat{HD}$
Then: $\overline{CR} \cong \overline{HD}$



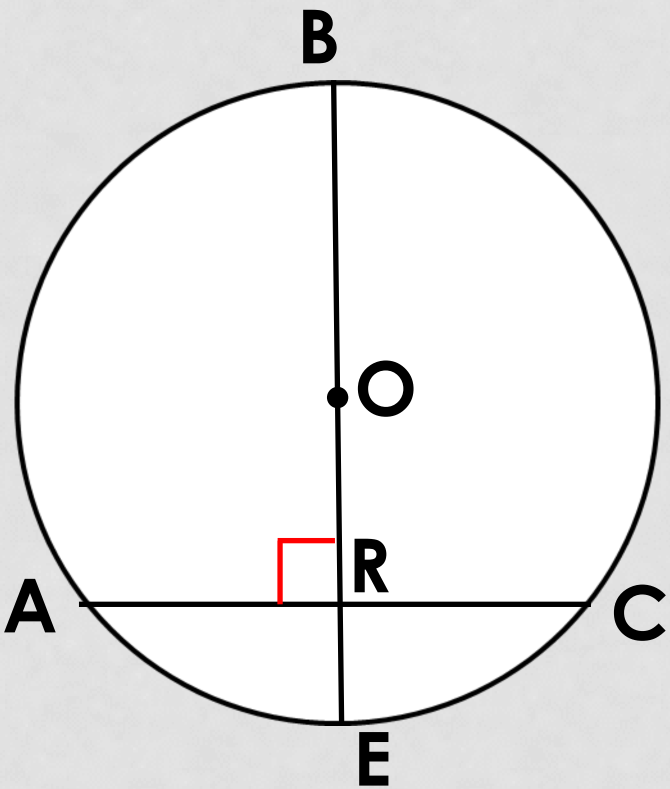
(2) If: $\overline{CR} \cong \overline{HD}$
Then: $m\,\widehat{CR} \cong m\,\widehat{HD}$

ARCS AND CHORDS

Theorem 9-5: A diameter that is perpendicular to a chord bisects the chord and its arc.

Given: $\odot 0$; $\overline{BE} \perp \overline{AC}$

Then: $\overline{AR} \cong \overline{RC}$; $\widehat{AE} \cong \widehat{CE}$

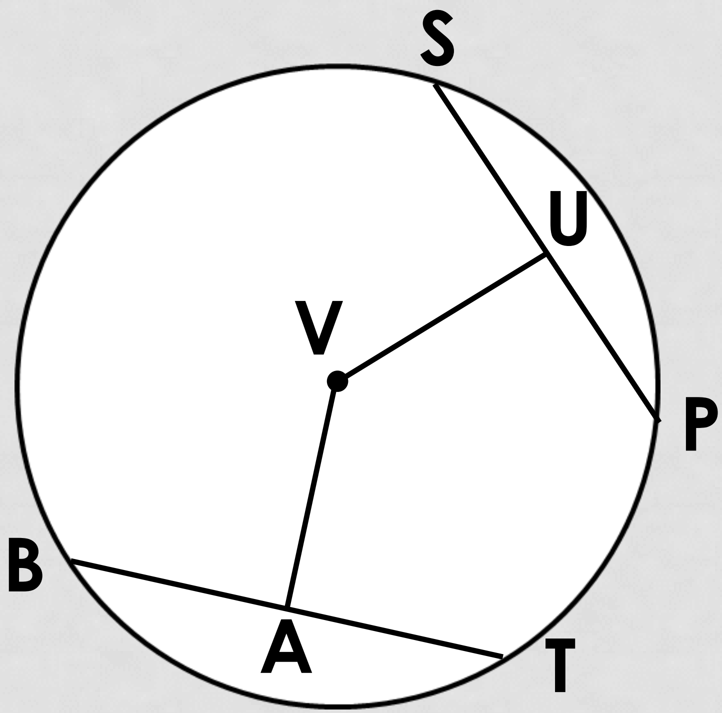


ARCS AND CHORDS

Theorem 9-6: In the same circle, or in congruent circles:

(1) Chords equally distant from the center are congruent;

(2) Congruent chords are equally distant from the center.



With $\overline{AV}$ and $\overline{UV}$ as the respective distances between $\overline{BT}$ and $\overline{SP}$;

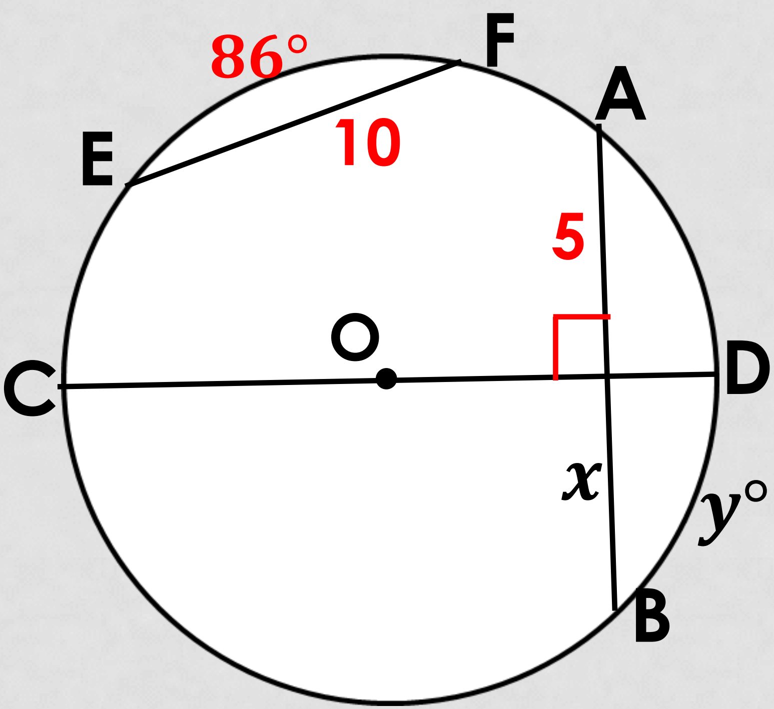
(1) If $AV = UV$
Then $\overline{BT} \cong \overline{SP}$

(2) If $\overline{BT} \cong \overline{SP}$
Then $AV = UV$

PRACTICE USING THE THEOREMS

- Find the measures of x and y.

1.)



Diameter $\overline{CD}$ bisects chord $\overline{AB}$, thus

$$x = 5$$

By theorem 9-5

$\overline{AB} \cong \overline{EF}$, so $\mathbf{m}\,\widehat{AB} = 86$

By theorem 9-4

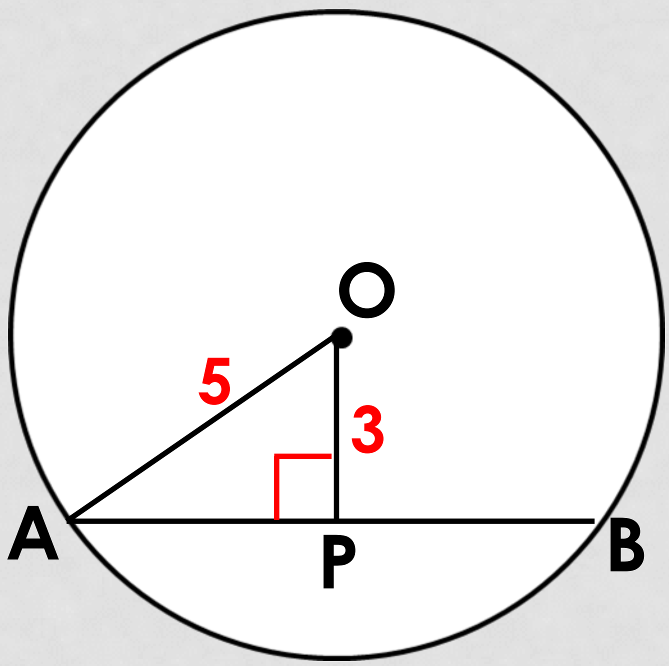
Diameter $\overline{CD}$ bisects chord $\overline{AB}$, so

$$y = 43$$

By theorem 9-5

PRACTICE USING THE THEOREMS

2.) Find the length of chord $\overline{AB}$



Using a Pythagorean triple, we can show that

$$\boldsymbol{AP = 4}$$

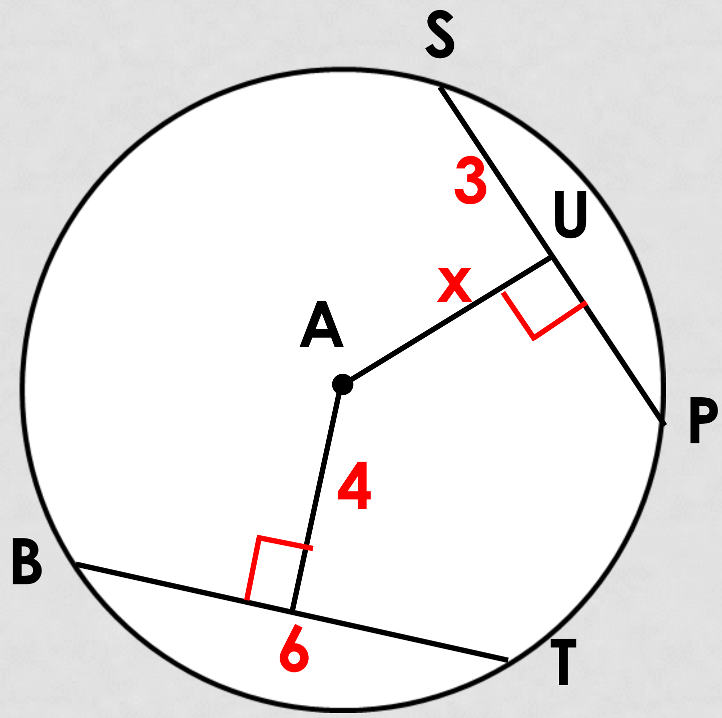
Also, $\overline{OP}$ bisects $\overline{AB}$, thus

$$\boldsymbol{AB = 8}$$

By theorem 9-5

PRACTICE THE RULES

- Find the value of x.



$\overline{AU}$ bisects $\overline{SP}$, thus

$$SP = 6$$

By theorem 9-5

This makes $\overline{SP} \cong \overline{BT}$

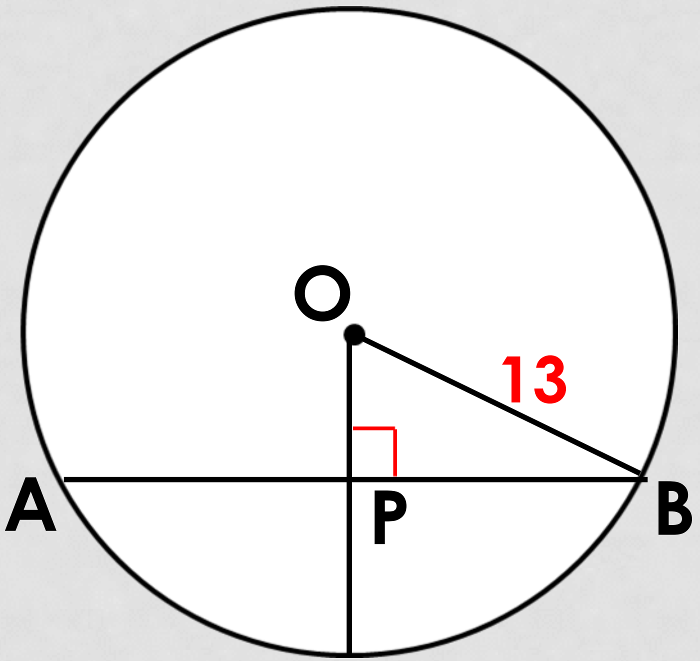
Thus

$$x = 4$$

By theorem 9-6

FINAL PRACTICE

- Find the measure(s) given.



Find OP if $AB = 24$

Since $AB = 24$, then

$$PB = 12$$

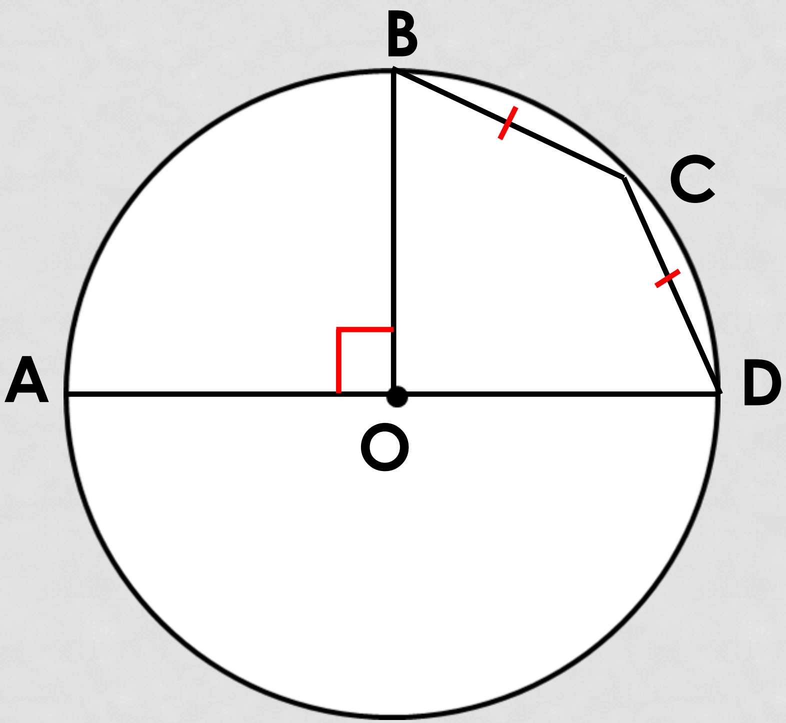
By Theorem 9-5

You can use the Pythagorean Theorem, or a triple, which will result in

$$OP = 5$$

FINAL PRACTICE

- Find the measure(s) given.



Find m $\widehat{CD}$

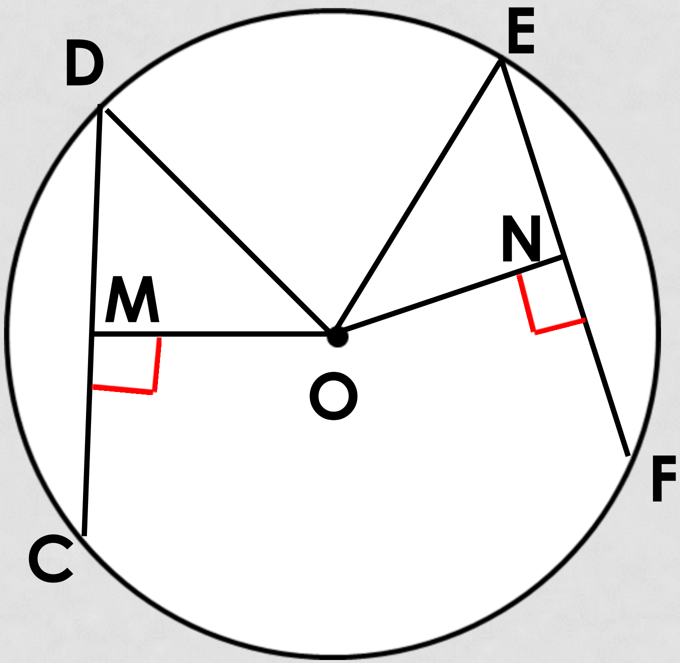
Since $\overline{BC} \cong \overline{CD}$, then their arcs must also be $\cong$, by theorem 9-4.

Since $m\,\widehat{BD} = 90$, then

$$m\,\widehat{CD} = \frac{1}{2} \times 90$$
$$= 45$$

FINAL PRACTICE – CHALLENGE PROBLEM

- Find the measure(s) given.



If $OM = ON = 7$ and $CM = 6$, find

- $\boldsymbol{DM} = \mathbf{6}$
- $\boldsymbol{EF} = \mathbf{12}$
- $\boldsymbol{DO} = \sqrt{\mathbf{85}}$
- $\boldsymbol{EO} = \sqrt{\mathbf{85}}$