Geometry Unit 9

9-5 Inscribed Angles in Circles

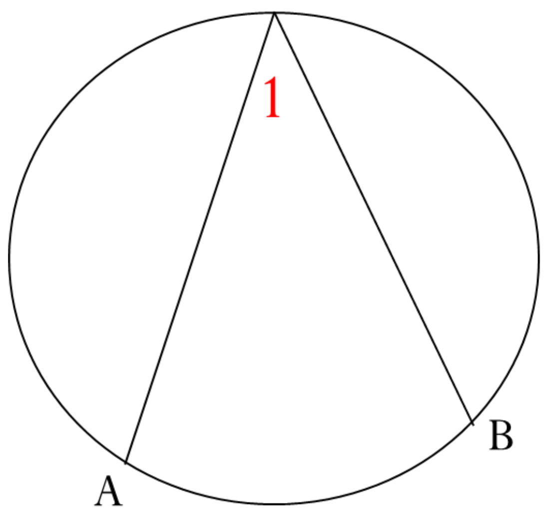
Inscribed Angles in Circles

- **Content Objective**: Students will be able to identify inscribed angles and their intercepted arcs in circles.

- **Language Objective**: Students will be able to solve for the missing measures of inscribed angles and their intercepted arcs in a variety of problems.

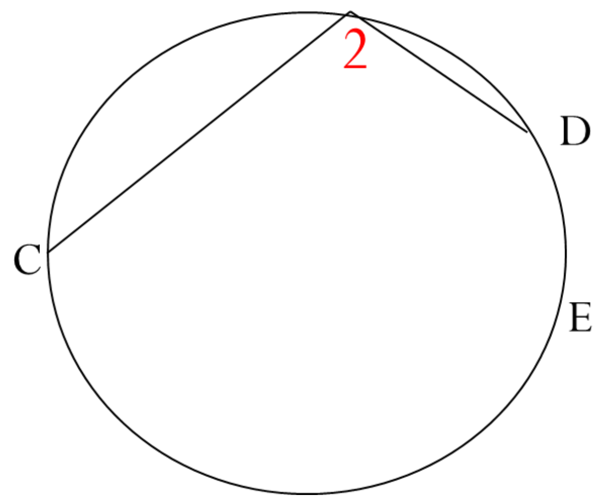
Inscribed Angles in a Circle

- An **Inscribed Angle** is an angle whose vertex is on a circle and whose sides are chords of the circle.
- The arc created by the chords is known as the **Intercepted Arc**.



Intercepted Arc

< 1 is the Inscribed Angle
$\widehat{AB}$ is the Intercepted Arc

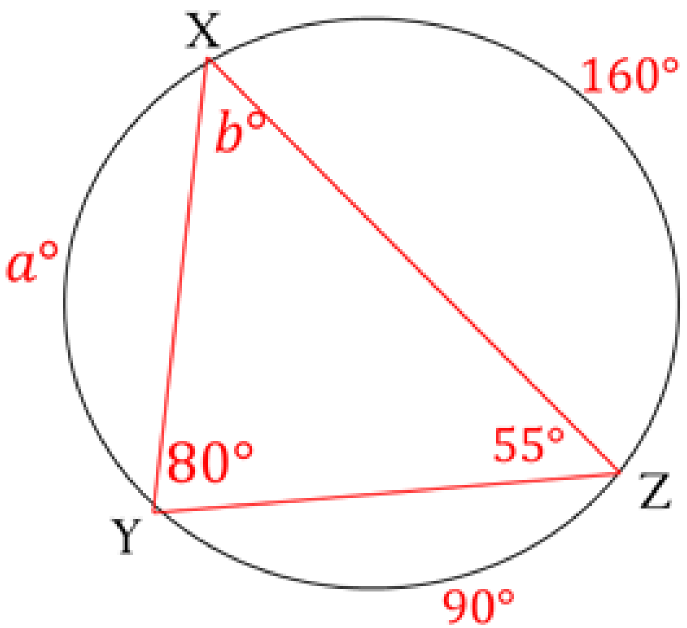


Intercepted Arc

< 2 is the Inscribed Angle
$\widehat{CED}$ is the Intercepted Arc

Warm-up: 9-5 Practice

1.) Name each Inscribed Angle, along with its intercepted arc?



Angle	Intercepted Arc
$< X$	$\widehat{YZ}$
$< Y$	$\widehat{XZ}$
$< Z$	$\widehat{XY}$

2.) What do you notice about the measure of $< Y$ and the measure of its Intercepted Arc?

Theorems For Inscribed Angles

- **Theorem 9-7**: The measure of an inscribed angle is equal to half the measure of its intercepted arc.

Equation:

$$\textbf{Inscribed Angle} = \frac{\mathbf{1}}{\mathbf{2}} \times \textbf{Intercepted Arc}$$

Example:

If $m\ \widehat{AC} = 120°$

Then $m < ABC = \frac{1}{2} m\ \widehat{AC}$

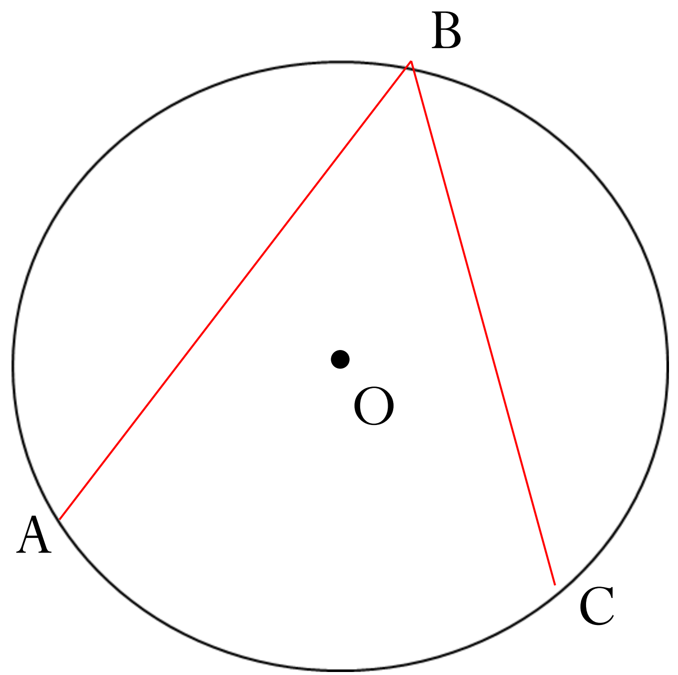
$$= \frac{1}{2} \times 120 = \mathbf{60°}$$

Example:

If $m < ABC = 55°$

Then $m\ \widehat{AC} = 2 \times m < ABC$

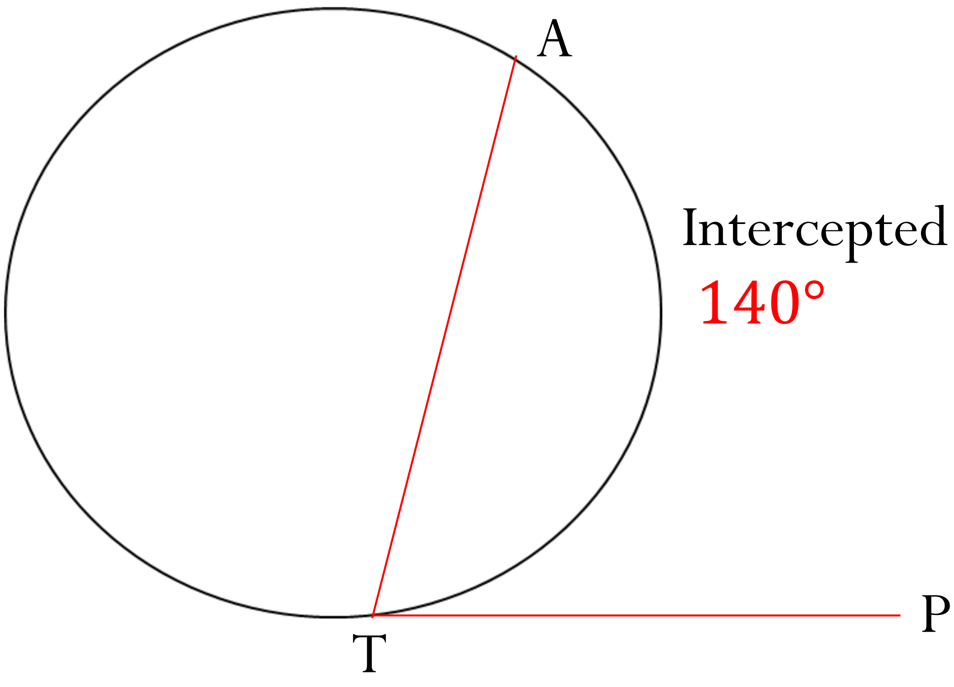
$$= 2 \times 55 = \mathbf{110°}$$



Intercepted Arc

Theorems For Inscribed Angles

- **<u>Theorem 9-8</u>**: The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.



If: $\overline{TP}$ is a tangent

$\widehat{AT}$ is the intercepted arc

Then: $m < ATP = \frac{1}{2}\widehat{AT}$

Example:

If $m\ \widehat{AT} = 140°$

$$m < ATP = \frac{1}{2}\widehat{AT}$$

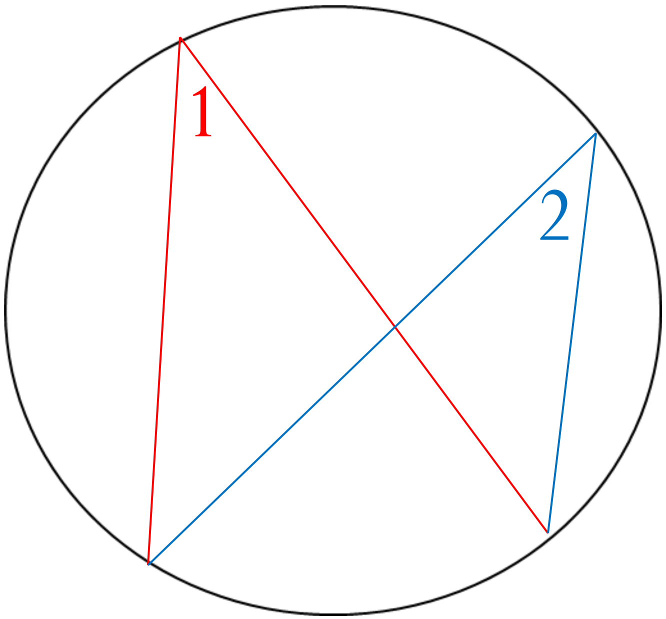
$$= \frac{1}{2} \times 140$$

$$= \mathbf{70°}$$

Corollaries For Inscribed Angles

- Corollary 1: If two inscribed angles intercept the same arc, then the angles are congruent.

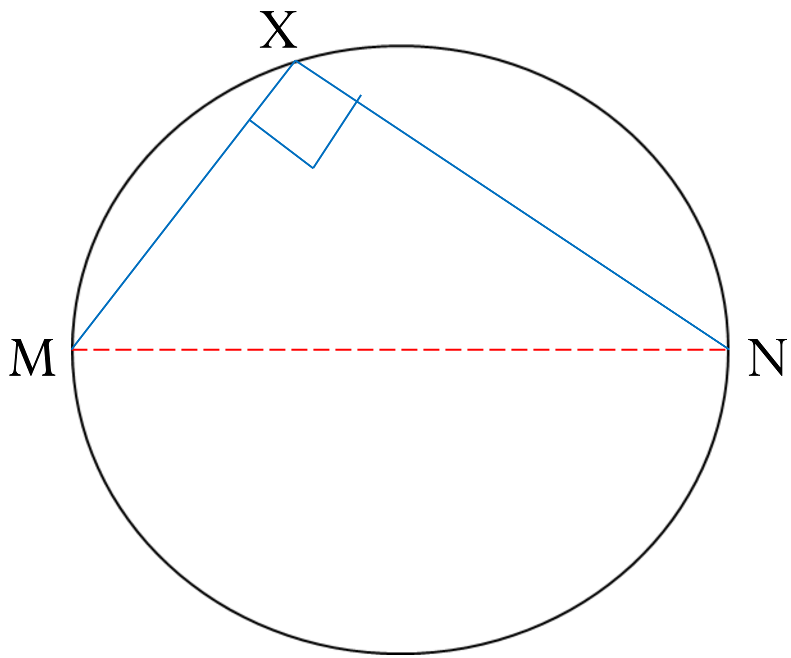
$$\angle 1 \cong \angle 2$$



Intercepted Arc

Corollaries For Inscribed Angles

- Corollary 2: An angle inscribed in a semicircle is a right angle.

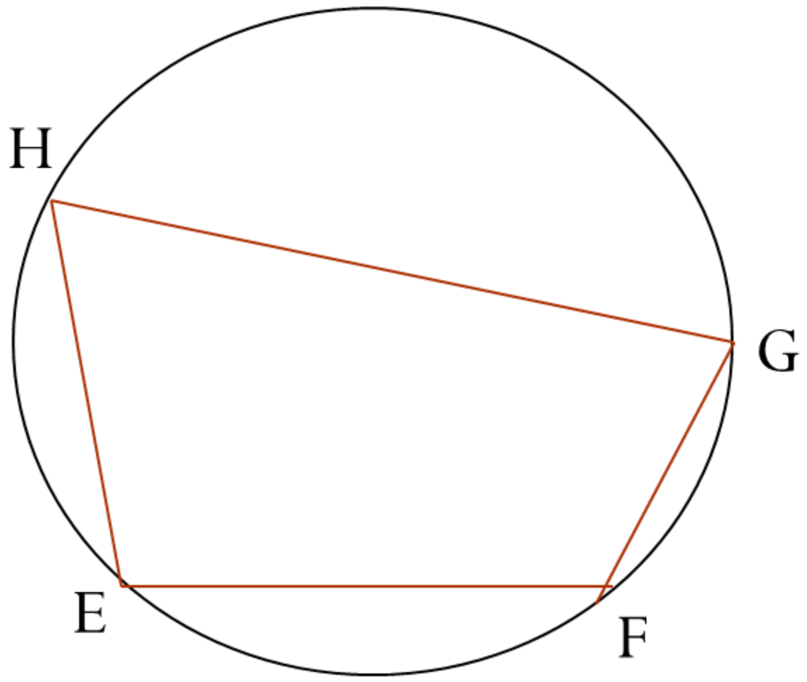


If: $\widehat{MXN}$ is a semicircle.

Then: $< X$ is a right angle.

Corollaries For Inscribed Angles

- Corollary 3: If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



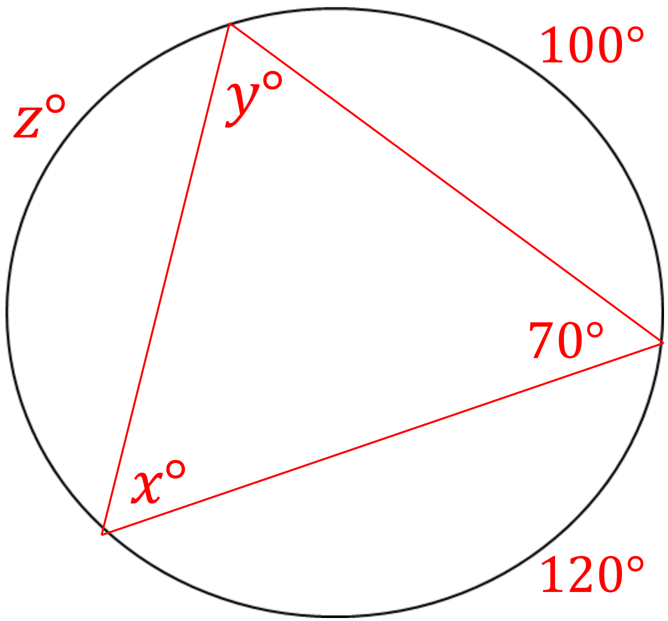
Given: Quad HEFG

$< H$ is supp. to $< F$

$< E$ is supp. to $< G$

Final Practice

- Use the theorems and corollaries given to solve for the variables given.



For x:

$$x = \frac{1}{2} \times 100$$

$$= \mathbf{50°}$$

For y:

$$y = \frac{1}{2} \times 120$$

$$= \mathbf{60°}$$

For z:

$$70 = \frac{1}{2} \times z$$

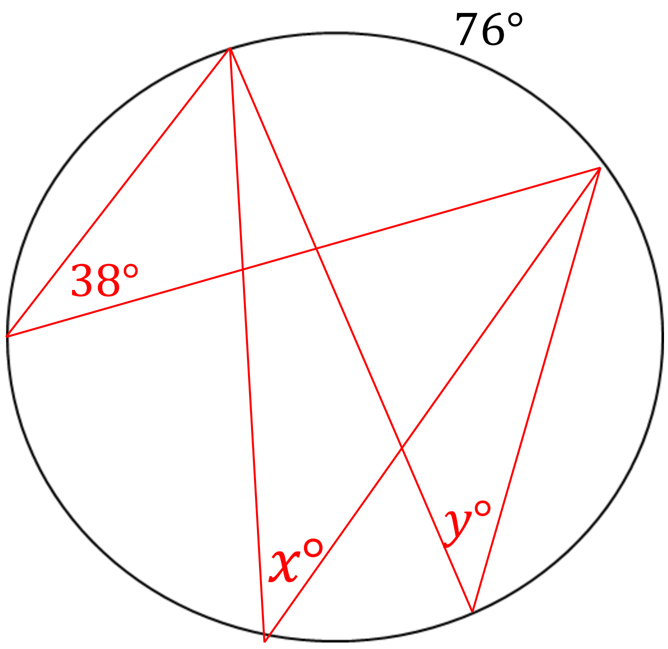
$$z = 70 \times 2$$

$$= \mathbf{140°}$$

Final Practice

- Use the theorems and corollaries given to solve for the variables given.

This will use Corollary 1



<u>For x</u>:

$$x = \frac{1}{2} \times 76$$

$$= \mathbf{38°}$$

<u>For y</u>:

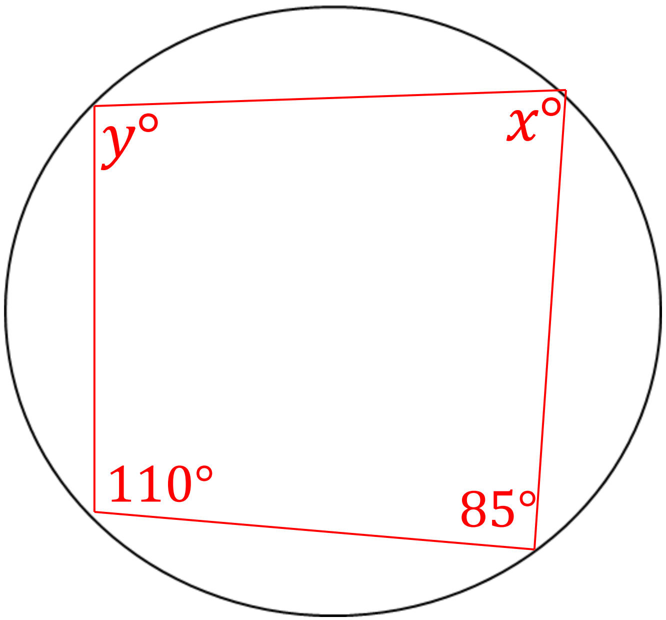
$$y = \frac{1}{2} \times 76$$

$$= \mathbf{38°}$$

Final Practice

- Use the theorems and corollaries given to solve for the variables given.

This will use Corollary 3



For x:

$$110 + x = 180$$

$$\boldsymbol{x = 70°}$$

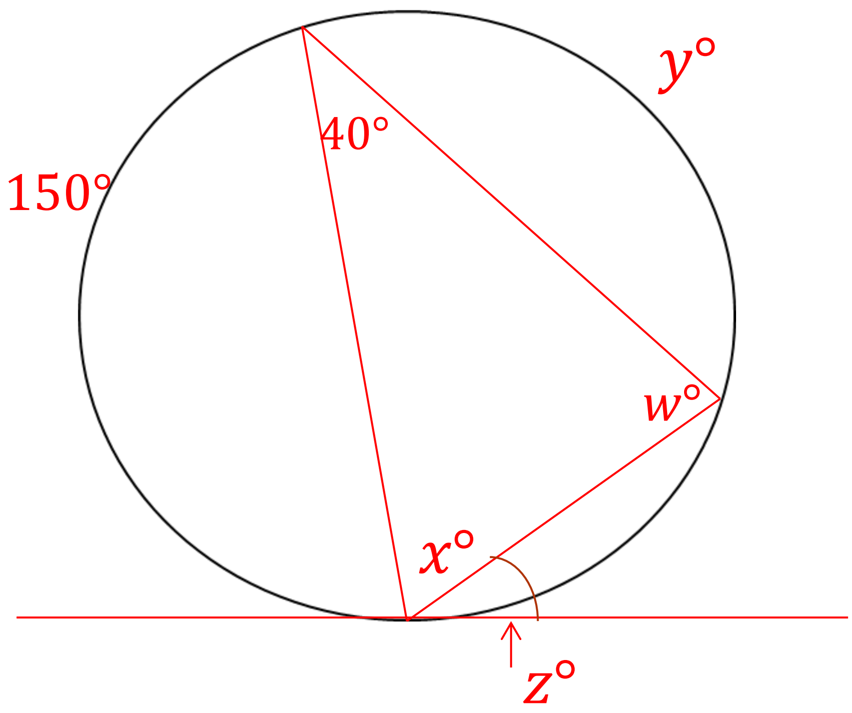
For y:

$$85 + y = 180$$

$$\boldsymbol{y = 95°}$$

Final Practice

- Use the theorems and corollaries given to solve for the variables given.



For w:

$$w = \frac{1}{2} \times 150$$

$$\boldsymbol{w = 75°}$$

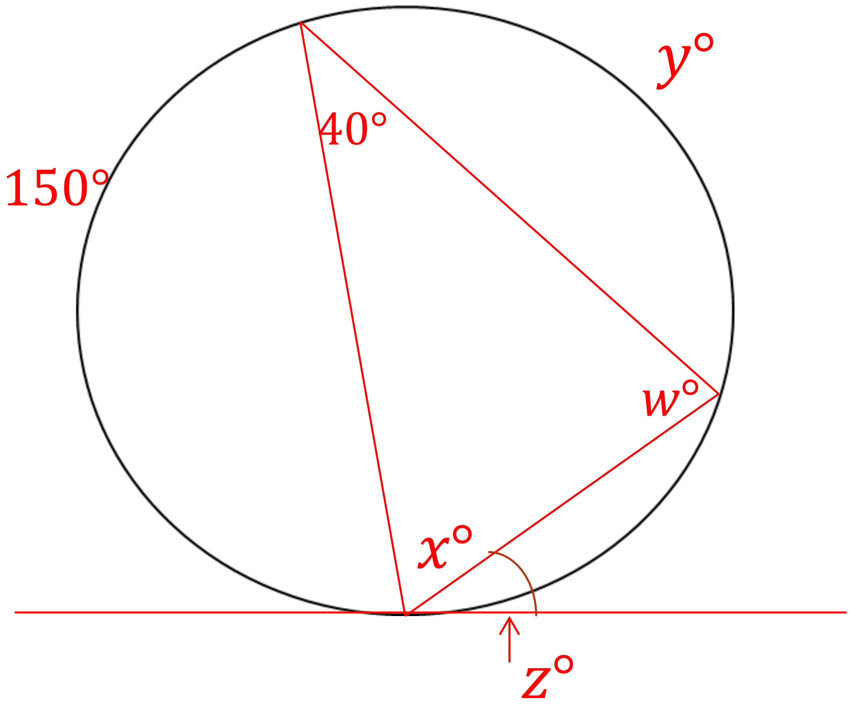
For x:

$$x = 180 - 40 - 75$$

$$\boldsymbol{x = 65°}$$

Final Practice

- Use the theorems and corollaries given to solve for the variables given.



For y:

$$y = 2 \times 65$$

$$\boldsymbol{w = 130°}$$

For z:

$$Intercepted\ Arc$$
$$= 360 - 150 - 130$$
$$= 80$$

$$z = \frac{1}{2} \times 80$$

$$\boldsymbol{z = 40°}$$