Geometry: Unit 1: Transformations

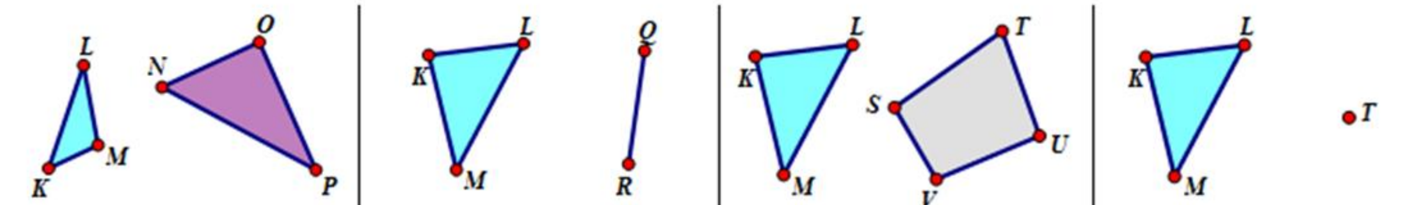
Chapter 14 (In Textbook)

Transformations

- **Objective**: Students will be able to do the following, regarding geometric transformations.
  - Write Transformations Symbolically and justify their choice.
  - Explain the movement of points for a given transformation.
  - Draw an image under each transformation.

Mapping

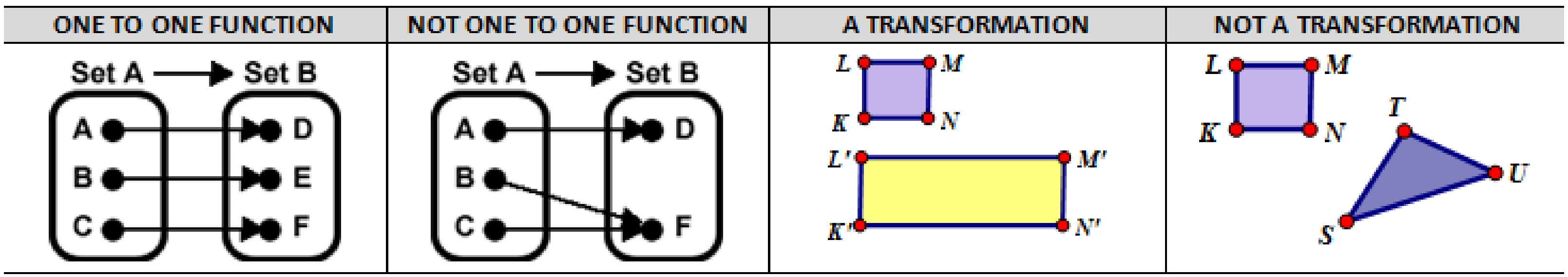
A correspondence between the pre-image and image is a **MAPPING IF AND ONLY IF** each member of the pre-image corresponds to one and only one member of the image.



#1 MAPPING	#2 MAPPING	#3 NOT A MAPPING	#4 MAPPING
If the pre-image is ΔKLM, K maps to N L maps to O M maps to P	If the pre-image is ΔKLM, M maps to R K maps to Q L maps to Q	If the pre-image is ΔKLM, K maps to S L maps to T **M maps to U** **M maps to V**	If the pre-image is ΔKLM, K maps to T L maps to T M maps to T

Transformation

- In Algebra when there is exactly the same number of elements in the domain as there is in the range it is called a **ONE TO ONE FUNCTION**.
- In geometry, when you have the same number of points in the pre-image as in the image, it is called a **TRANSFORMATION**.



Transformations: Image and Pre-Image

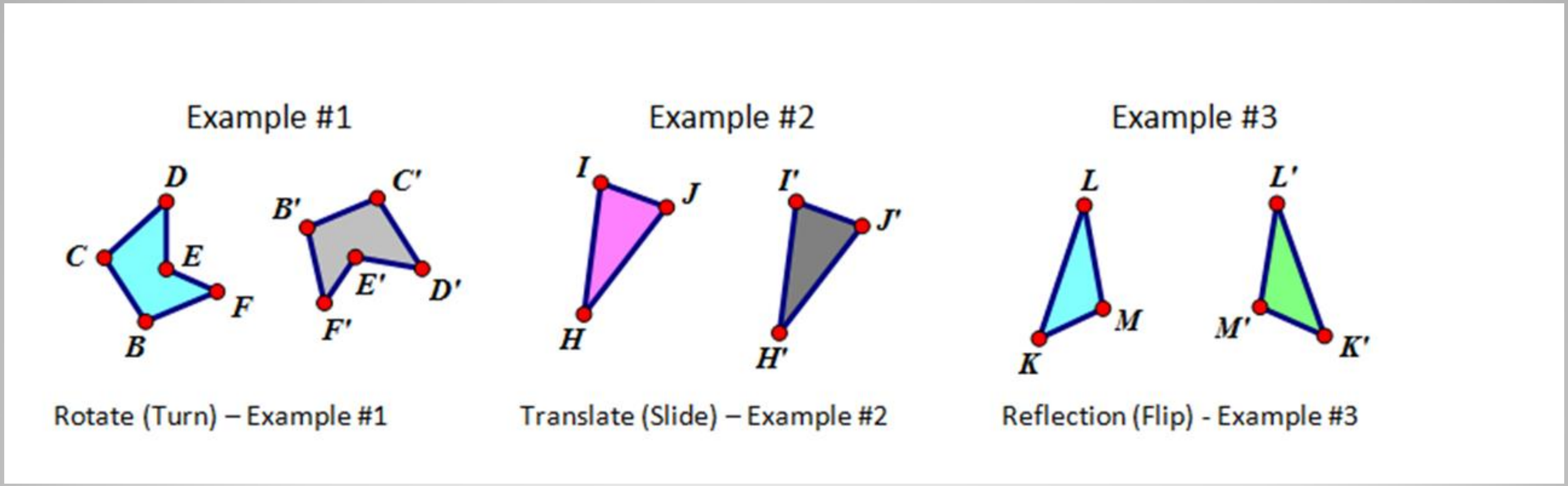
A **transformation** is a one-to-one correspondence between the points of the **pre-image** and the points of the **image**. A **transformation** guarantees that if our **pre-image** has three points, then our **image** will also have three points.

Pre-Image: The figure prior to transformation (***P***)

Image: The figure after the transformation (***P′***)

Isometry

An ISOMETRIC TRANSFORMATION (RIGID MOTION) is a transformation that preserves the distances and/or angles between the pre-image and image.



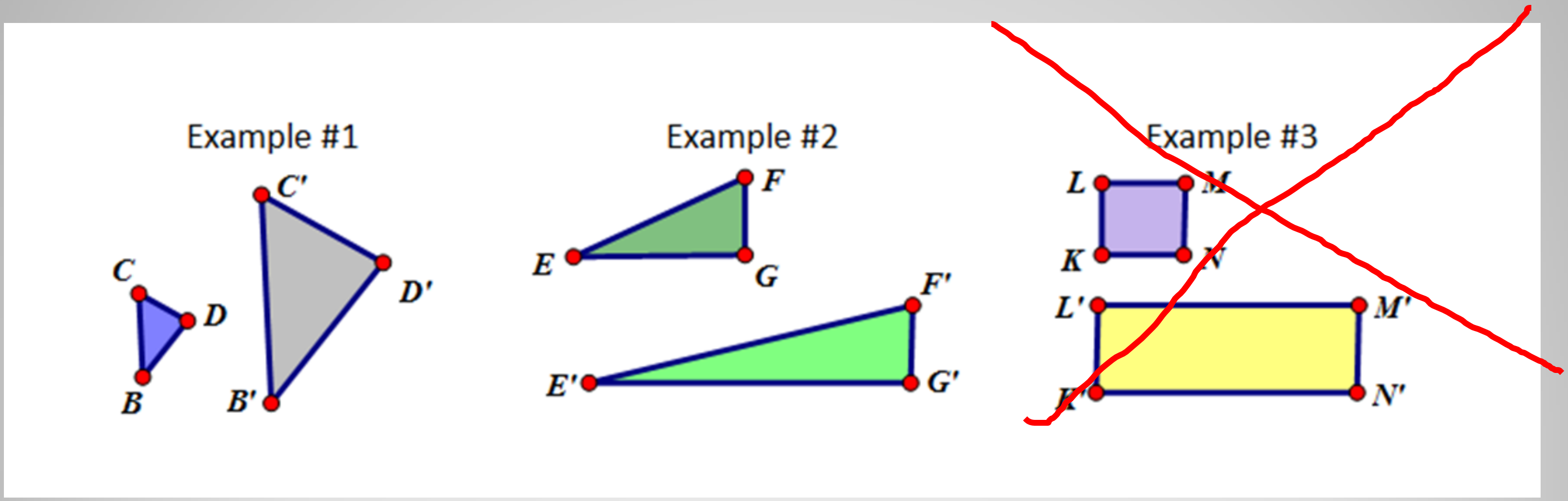
Isometry

- An Isometric Transformation has the following properties are preserved:
  - Distance (All lengths stay the same)
  - Angle measure (All angles stay the same)
  - Parallelism (All lines that are parallel stay parallel)
  - Collinearity (All points on a line remain on a line)

- In short, the transformed figure (**Image**) is the same shape and size as the original figure **(Pre-Image).**

Non-Isometry

- A **NON-ISOMETRIC TRANSFORMATION (NON-RIGID MOTION)** is a transformation that does not preserve the distances between the pre-image and image.



Non-Isometry

- A Non-Isometric Transformation has the following properties preserved:
  - Angle measure (All angles stay the same)
  - Parallelism (All lines that are parallel stay parallel)
  - Collinearity (All points on a line remain on a line)
- In short, the transformed figure (**Image**) has the same shape as the original figure (**Pre-Image**), but not the same size.

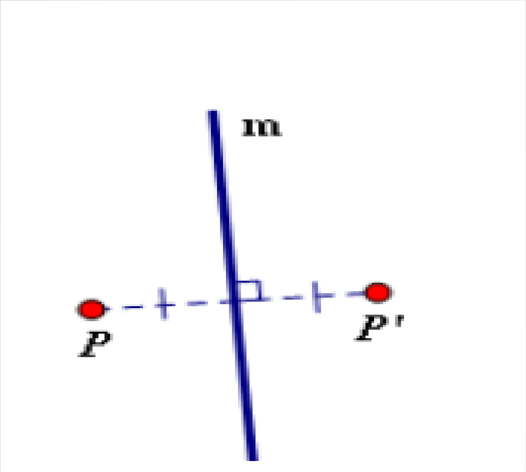
Transformations

- The following Transformations are Isometries:
  - Reflections
  - Rotations
  - Translations
- The following Transformations are Non-Isometries:
  - Dilations

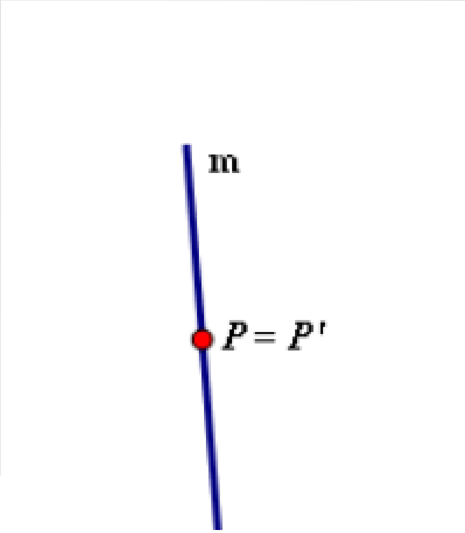
Reflections (Textbook pg. 577)

A **reflection** in a line *m* is an isometric transformation that maps a point *P* on the plane to a point *P'*, so that the following properties are true:

- 1. If P is not on the line *m*, then the line *m* is a perpendicular bisector of $\overline{PP'}$.

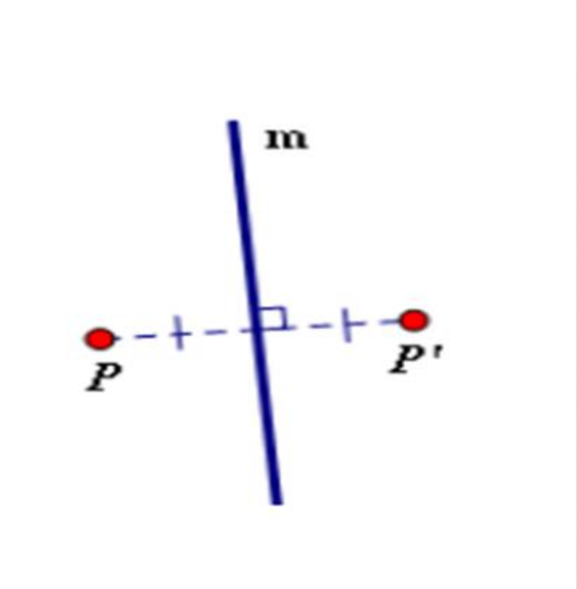


- 2. If P is on the line *m*, then $P = P'$.



REFLECTIONS: NOTATION

- To abbreviate a reflection in the line *m*, we write R_m. To abbreviate the statement R_m *maps P to P'*, we write $R_m: P \to P'$ or $R_m(P) = P'$.



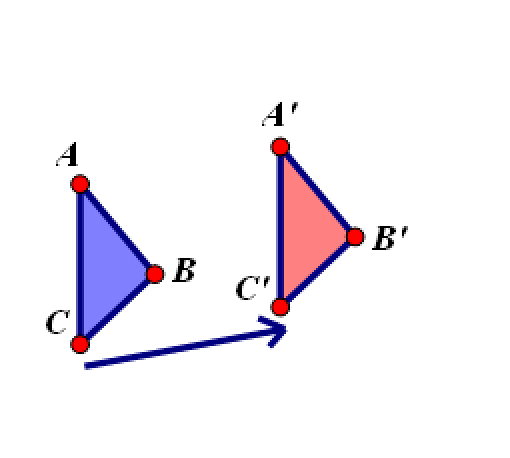
TRANSLATIONS (TEXTBOOK PG. 583)

- A transformation that glides all points of the plane the same distance in the same direction is called a **translation.**

- When working on the coordinate plane, a vector is used to describe the fixed distance and the given direction often denoted by **<x,y>.** The x value describes the effect on the x coordinates (right or left) and the y value describes the effect on the y coordinates (up or down).

The pre-image and image have the same shape and size.

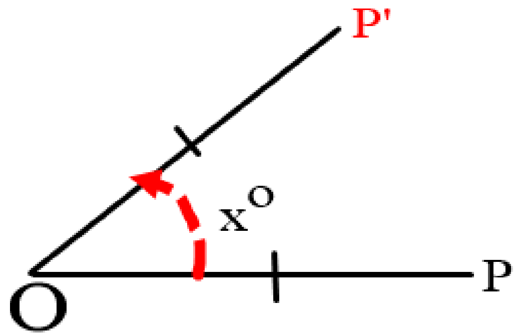
$$T_{<x,y>}(\Delta ABC) = \Delta A'B'C'$$



Rotations (Textbook pg. 588)

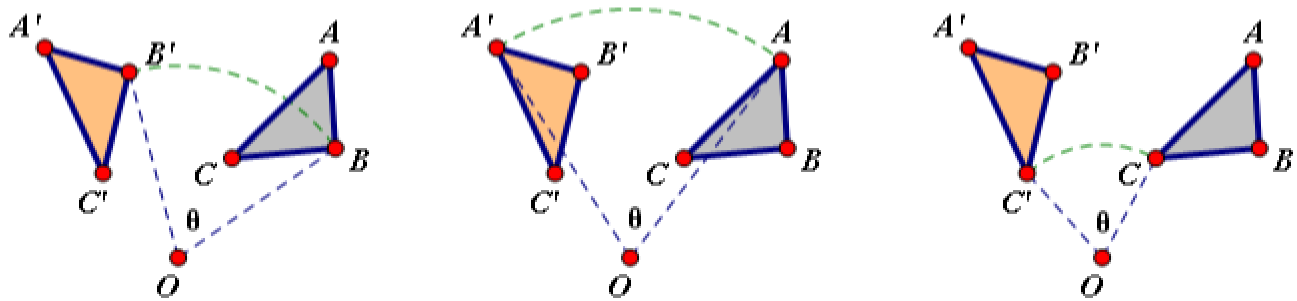
- A **rotation** is an isometric transformation that turns a figure about a fixed point called the center of rotation. Rays drawn from the center of rotation to a point and its image form an angle called the angle of rotation.
- For a counterclockwise rotation about a point O through x°, we write $R_{(O,x)}$. A counterclockwise rotation is considered positive, and a clockwise rotation is considered negative.

$R_{(O,\,x^{o})}$



Rotations

- An object and its rotation are the same shape and size, but the figures may be turned in different directions.



Final Words

- For the next few days, there will be a sub.
- Follow the subs rules.
- Be on your best behavior.
- Bring your textbooks the rest of this week.