GEOMETRY UNIT 12

SLOPE AND MIDPOINT

WARM-UP

Find the Slope of the line through the following pairs of points.

1. $(7, 2)$ **and** $(2, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 2}{2 - 7}$$

$$= -\frac{5}{5} = -1$$

2. $(0, 0)$ **and** $(5, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{5 - 0}$$

$$= \frac{1}{5}$$

3. $(1, 6)$ **and** $(4, 6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 6}{4 - 1}$$

$$= \frac{0}{3} = 0$$

4. $(3, 3)$ **and** $(3, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{3 - 3}$$

$$= \frac{4}{0} = ?$$

Slope is undefined

SLOPE AND MIDPOINT

- **Content Objective:** Students will be able to identify the slopes and midpoints of lines.

- **Language Objective:** Students will be able to calculate the slope and midpoint of a line given two points.

SLOPE: RECAP

- The **Slope** of a line is the ratio of *change in y*

(vertical change, or *rise*) to the *change in x*

(horizontal change, or *run*).

- Parallel Lines have slopes that are **Equal**
- Perpendicular Lines have slopes that **have a product of -1**
- Positive slopes(#2 on the warm-up) **rise** to the right.
- Negative slopes (#1 on the warm-up) **fall** to the right.

SLOPE: CONTINUED

- From the warm-up, you noticed something about the answers for problems 3 and 4.

3. $(1, 6)$ **and** $(4, 6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 6}{4 - 1}$$

$$= \frac{0}{3} = 0$$

4. $(3, 3)$ **and** $(3, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{3 - 3}$$

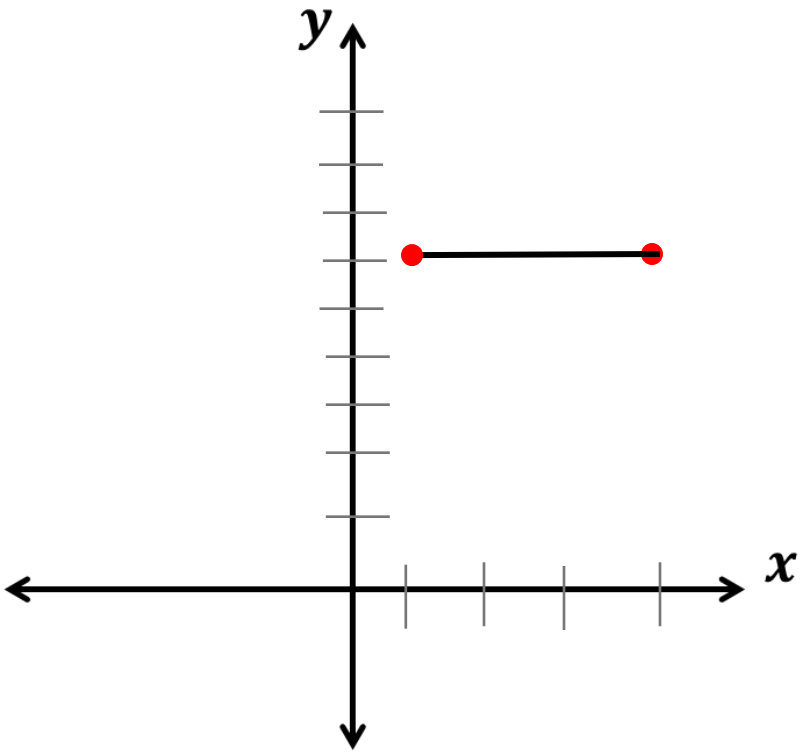
$$= \frac{4}{0} = ?$$

Slope is undefined

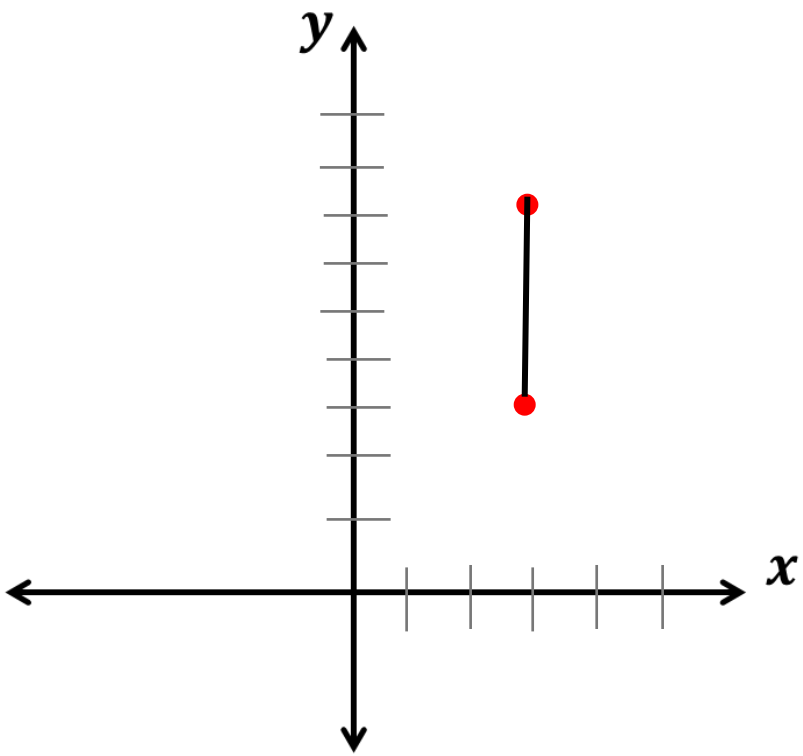
On the two graphs provided, graph the points given of these two problems. Name the kind of line the points make.

SLOPE: CONTINUED

3.) $(1, 6)$ **and** $(4, 6)$



4.) $(3, 3)$ **and** $(3, 7)$



Conclusion: Horizontal lines have a slope of **0** and Vertical lines have an undefined slope.

MIDPOINT

- As a reminder, the **Midpoint** of a line segment is a point M such that $AM = MB$

- We can calculate the value of this midpoint if we have the values of the endpoints.
- Ex: If $A = x_1$ and $B = x_2$, then the value of M will be

$$M = \frac{x_1 + x_2}{2}$$

The average of the values of A and B

Key Question: Could this idea also be used to find the midpoint of two points on the (x,y) – coordinate plane?

THE MIDPOINT FORMULA

Theorem 13-5: The midpoint of the segment that joins points (x_1, y_1) and (x_2, y_2) is the point

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 1: Find the midpoint of the segment that joins $(-11,3)$ and $(8,-7)$.

Solution:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{-11+8}{2}, \frac{3+(-7)}{2}\right)$$

$$M = \left(\frac{-3}{2}, \frac{-4}{2}\right) = \left(\mathbf{\frac{-3}{2}}, \mathbf{-2}\right)$$

PRACTICE

Find the midpoint of the segment that joins the points given.

2.) $(2,1)$ and $(8,-5)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{2+8}{2}, \frac{1+(-5)}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{-4}{2}\right) = (\mathbf{5}, \mathbf{-2})$$

3.) $(1,-3)$ and $(5,1)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{1+5}{2}, \frac{-3+1}{2}\right)$$

$$M = \left(\frac{6}{2}, \frac{-2}{2}\right) = (\mathbf{3}, \mathbf{-1})$$

OTHER USE FOR MIDPOINT

Example: ***M*** is the midpoint of $\overline{\boldsymbol{AB}}$, where the coordinates of ***A*** are given. Find the coordinates of ***B***

A: $(1, -3)$; M: $(5,1)$

Solution: From the equation, we have

$$(5,1) = \left(\frac{1+x}{2}, \frac{-3+y}{2}\right)$$

Separate to solve for ***x*** and ***y.***

For x:

$$5 = \frac{1+x}{2}$$

$$10 = 1 + x$$

$$\boldsymbol{x = 9}$$

For y:

$$1 = \frac{-3+y}{2}$$

$$2 = -3 + y$$

$$\boldsymbol{y = 5}$$

Thus, the coordinates of ***B*** are $(\mathbf{9}, \mathbf{5})$

GROUP PRACTICE – SLOPES AND MIDPOINT

Find the midpoint of the segment that joins the points given.

1.) $(3,5)$ and $(9,-7)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{3+9}{2}, \frac{5+(-7)}{2}\right)$$

$$M = \left(\frac{12}{2}, \frac{-2}{2}\right) = \mathbf{(6,-1)}$$

2.) $(2,5)$ and $(-1,2)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{2+(-1)}{2}, \frac{5+2}{2}\right)$$

$$M = \left(\mathbf{\frac{-1}{2}}, \mathbf{\frac{7}{2}}\right)$$

GROUP PRACTICE – SLOPES AND MIDPOINT

Find the midpoint of the segment that joins the points given.

3.) $(0,4)$ and $(4,3)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{0+4}{2}, \frac{4+3}{2}\right)$$

$$M = \left(\frac{4}{2}, \frac{7}{2}\right) = \left(\mathbf{2}, \frac{\mathbf{7}}{\mathbf{2}}\right)$$

GROUP PRACTICE – SLOPES AND MIDPOINT

Find the slope and midpoint of the segment that joins the points given.

4.) $(3, -8)$ and $(-5,2)$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - (-8)}{-5 - 3}$$

$$m = \frac{10}{-8} = -\frac{\mathbf{5}}{\mathbf{4}}$$

Midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{3 + (-5)}{2}, \frac{-8 + 2}{2}\right)$$

$$M = \left(\frac{-2}{2}, \frac{-6}{2}\right) = (\mathbf{-1}, \mathbf{-3})$$

GROUP PRACTICE – SLOPES AND MIDPOINT

Find the slope and midpoint of the segment that joins the points given.

5.) $(-3,4)$ and $(7,8)$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8 - 4}{7 - (-3)}$$

$$m = \frac{4}{10} = \mathbf{\frac{2}{5}}$$

Midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-3 + 7}{2}, \frac{4 + 8}{2}\right)$$

$$M = \left(\frac{4}{2}, \frac{12}{2}\right) = \mathbf{(2, 6)}$$

GROUP PRACTICE – SLOPES AND MIDPOINT

Find the slope and midpoint of the segment that joins the points given.

6.) $(-7,11)$ and $(1,-4)$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 11}{1 - (-7)}$$

$$m = \frac{\mathbf{-15}}{\mathbf{8}}$$

Midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-7 + 1}{2}, \frac{11 + (-4)}{2}\right)$$

$$M = \left(\frac{-6}{2}, \frac{7}{2}\right) = \left(\mathbf{-3}, \mathbf{\frac{7}{2}}\right)$$