GEOMETRY UNIT 12

Slope of a Line

WARM-UP: THEOREM FOR SIMILAR SOLIDS

Theorem 12-11: If the scale factor of two similar solids is $a : b$, then

(1) The ratio of corresponding perimeters is $\boldsymbol{a : b}$

(2) The ratio of the base areas, of the lateral area, and of the total areas is $\boldsymbol{a^2 : b^2}$

(3) The ratio of the volumes is $\boldsymbol{a^3 : b^3}$

WARM-UP FROM 12-5

1.) Given the following measurements for similar solids, identify the reduced ratio for each of the following.

Given height 4 and height 7

*When given heights/slant heights /radii /perimeter /circumference, you use a reduced ratio of the values given as your scale

(a.) Scale Factor ___$\mathbf{4/7}$___

(b.) Total Area ___$\mathbf{4^2/7^2 = 16/49}$___

WARM-UP FROM 12-5

- Given the following measurements for similar solids, identify the reduced ratio for each of the following.
- Given areas **18**π and 50π.
- Pay attention to the fact that it gives **AREAS**, which have an $\boldsymbol{a^2/b^2}$ relationship to the $\boldsymbol{a/b}$ scale factor
- To solve for the scale factor, first reduce the current ratio, then take the square root of both values to get the scale factor
- (a.) Scale Factor

  - $\mathbf{{}^{18\pi}/_{50\pi} = {}^{9}/_{25}}$
  - $\mathbf{{}^{\sqrt{9}}/_{\sqrt{25}} = {}^{3}/_{5}}$

(b.) Volume

$$\mathbf{{}^{3^3}/_{5^3} = {}^{27}/_{125}}$$

SLOPE OF A LINE

- **Content Objective**: Students will be able to identify slopes in straight lines

- **Language Objective**: Students will be able to calculate the slope of a line with two or more given points.

SLOPE

- The **Slope** of a line is the ratio of *change in y* (vertical change, or *rise*) to the *change in x* (horizontal change, or *run*).

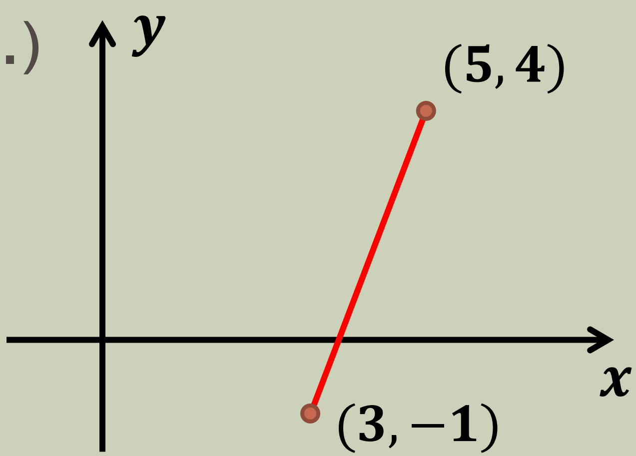
- Symbolically, the slope is denoted by an ***m.***

- Algebraically, the slope can be defined using the following equation, with points (x_1, y_1) and (x_2, y_2).

$$\boldsymbol{m = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}}$$

EXAMPLE: CALCULATE THE SLOPE OF EACH LINE

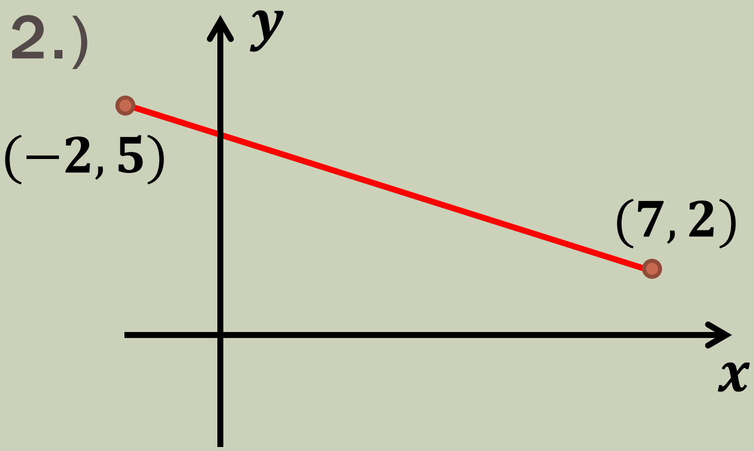
1.)



Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{5 - 3}$$
$$= \frac{5}{2}$$

2.)



Solution:

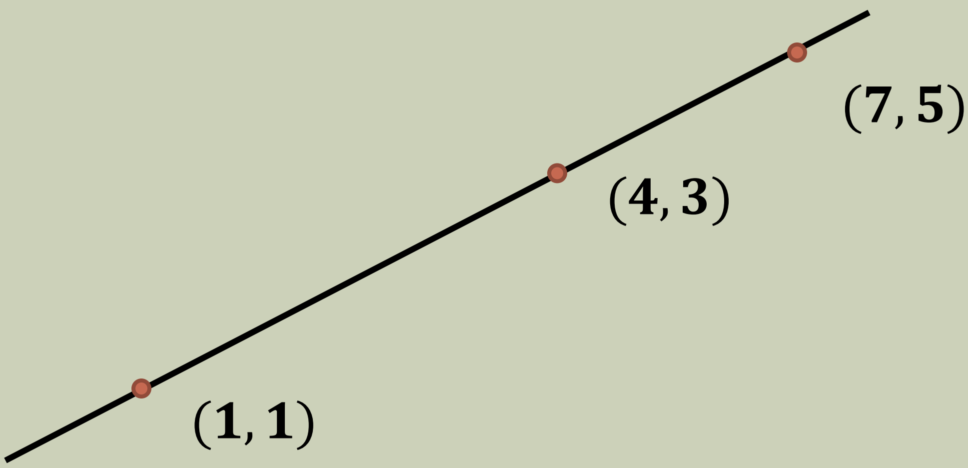
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{7 - (-2)}$$

$$= \frac{-3}{9} = -\frac{1}{3}$$

SLOPE

- When you are given several points on a line, you can use any two of them to compute the slope.

- **<u>Example</u>**: Find the slope of this line using every combination of pairs of points



<u>First combination:</u> (1,1) and (4,3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 1} = \frac{2}{3}$$

<u>Second combination:</u> (1,1) and (6,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{7 - 1} = \frac{4}{6} = \frac{2}{3}$$

<u>Third combination:</u> (4,3) and (7,5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{7 - 4} = \frac{2}{3}$$

SLOPES OF MULTIPLE LINES

- When looking for the slopes of two or more lines, we can find them in one of two ways (depending on the information given to us):

1.) Calculate the slopes of each individual line is only points are given.

2.) Use the relationship between the lines (if there is one) to get the slope of the other line(s) from a given slope.

If you recall, there are two major relationships between lines that we have discussed in the past:

- They are either __Parallel____ or _Perpendicular_.

PARALLEL LINES

- As a reminder, **Parallel Lines** (ll lines) are coplanar lines that do not intersect.

Notation: $\boldsymbol{l} \parallel \boldsymbol{n}$

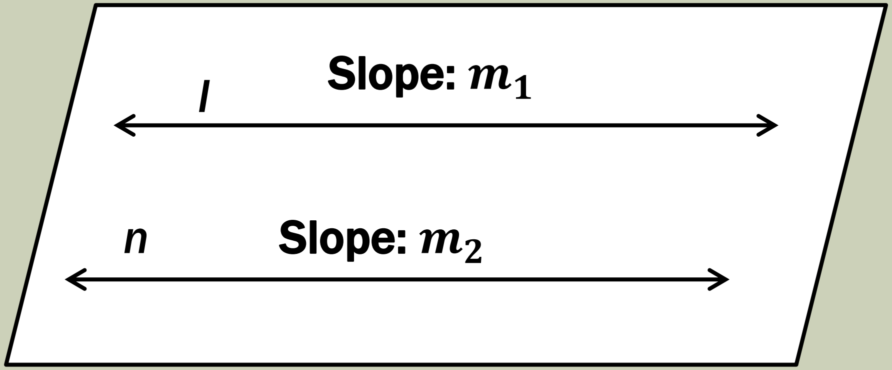
<u>Key Question</u>: From the image given, and from what you know about slopes, can you determine the relationship between the slopes of parallel lines? Discuss this question in your groups

SLOPES IN PARALLEL LINES

- **Theorem 13-3**: Two nonvertical lines are parallel if and only if their slopes are equal.

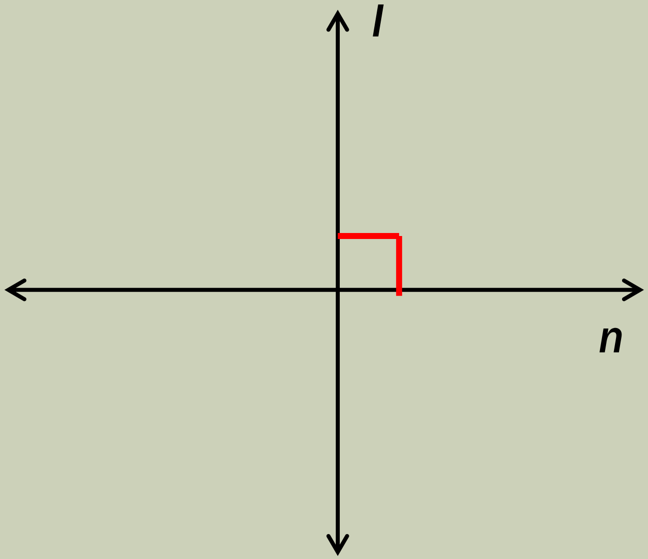
Given: Two nonvertical parallel lines, l and n, with slopes m_1 and m_2 respectively.

Then: $m_1 = m_2$



PERPENDICULAR LINES

- As a reminder, **Perpendicular Lines** ($\perp$ lines) are lines that intersect to form right angles.



Notation: $\boldsymbol{l} \perp \boldsymbol{n}$

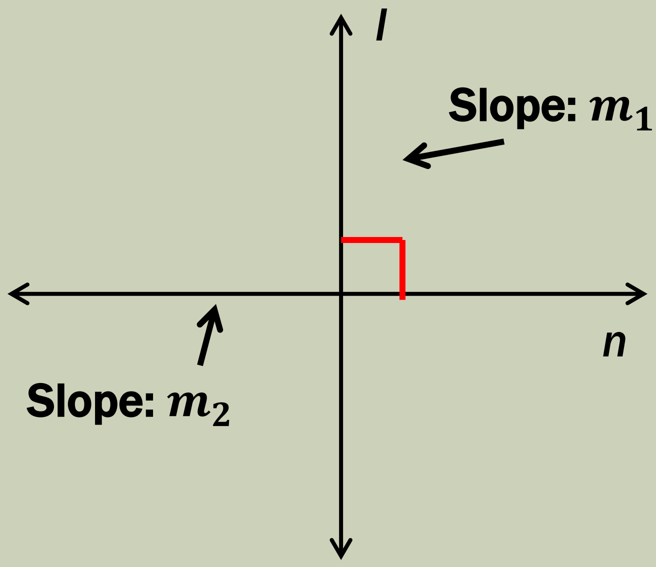
With $\boldsymbol{m < 1, m < 2, m < 3, m < 4 = 90°}$

<u>Key Question</u>: From the image given, and from what you know about slopes, can you determine the relationship between the slopes of perpendicular lines? Discuss this question in your groups

SLOPE IN PERPENDICULAR LINES

- **Theorem 13-4**: Two nonvertical lines are perpendicular if and only if **the product of their slopes is -1.**

Given: Two nonvertical perpendicular lines, l and n, with slopes m_1 and m_2, respectively.



Then: $m_1 \times m_2 = -1$

Or

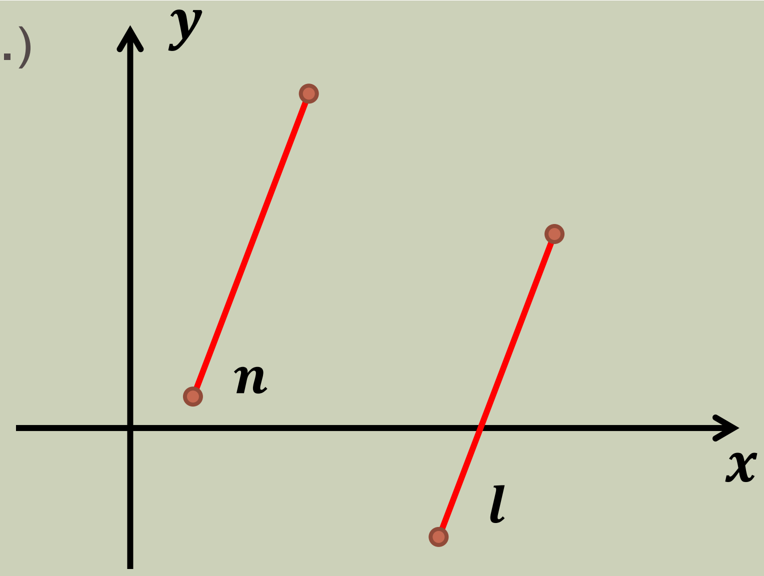
$$m_1 = \frac{-1}{m_2}$$

Or

$$m_2 = \frac{-1}{m_1}$$

PRACTICE: CALCULATE THE SLOPE OF EACH LINE

1.)



$Given: l \parallel n$

and

Line l has points $(4, -2)$ and $(5, 3)$

Slope of l, or m_1:

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{5-4}$$
$$= \frac{5}{1} = 5$$

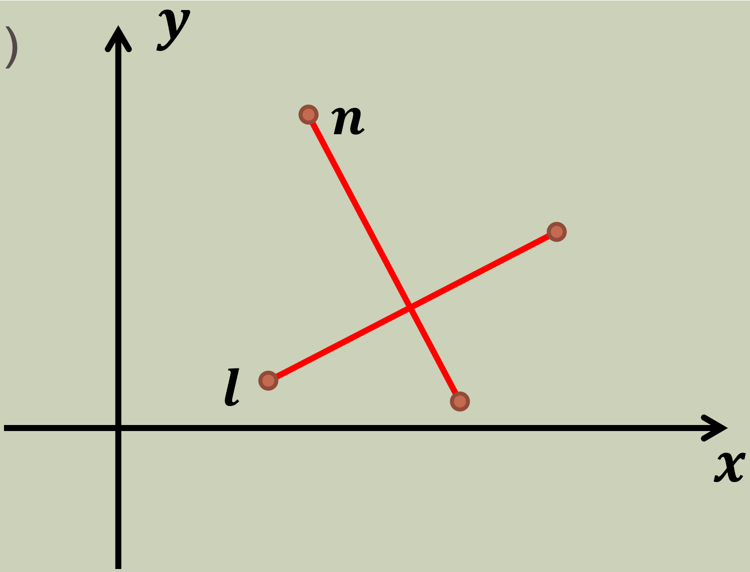
Slope of n, or m_2:

Since $l \parallel n$, then $m_1 = m_2$

Thus, $m_2 = 5$

PRACTICE: CALCULATE THE SLOPE OF EACH LINE

2.)



$Given: \boldsymbol{l} \perp \boldsymbol{n}$

and

Line l has points $(\mathbf{2}, \mathbf{1})$ and $(\mathbf{6}, \mathbf{3})$

Slope of l, or m_1:

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{6 - 2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

Slope of n, or m_2:

Since $\boldsymbol{l} \perp \boldsymbol{n}$, then $m_2 = \frac{-1}{m_1}$

Thus, $m_2 = \frac{-2}{1} = -2$

GROUP PRACTICE

- Complete the able of slope values:

Starting Points	Slope	Parallel Slope	Perpendicular Slope
$(1, 2)$ *and* $(-2, -5)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-5-2}{-2-1} = \frac{7}{3}$	$\frac{7}{3}$	$\frac{-3}{7}$
$(-4, 3)$ *and* $(6, -6)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-6-3}{6-(-4)} = \frac{-9}{10}$	$\frac{-9}{10}$	$\frac{10}{9}$