Geometry Unit 5

The Isosceles Triangle Theorems

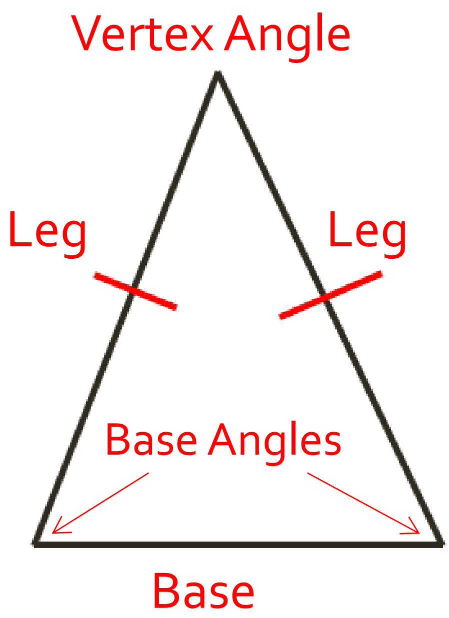
The Isosceles Triangle Theorems

- **Content Objective**: Students will be able to solve problems and proofs involving isosceles triangles.

- **Language Objective**: Students will be able to write equations for isosceles triangles, solving for variables.

Isosceles Triangles

- An Isosceles Triangle has the following properties
  - 2 Congruent Sides (known as the **legs**)
  - 1 Side with its own measure (known as the **base**)
  - The angle included between the legs is known as the **vertex angle**
  - Angles connected to the base are known as the **base angles**

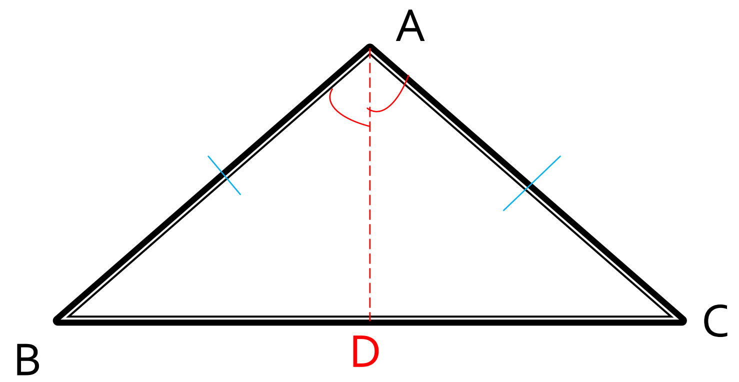


Theorem 4-1

- **<u>The Isosceles Triangle Theorem</u>:** Base angles of a isosceles triangle are congruent.

Given: $\overline{AB} \cong \overline{AC}$

Prove: $< B \cong < C$

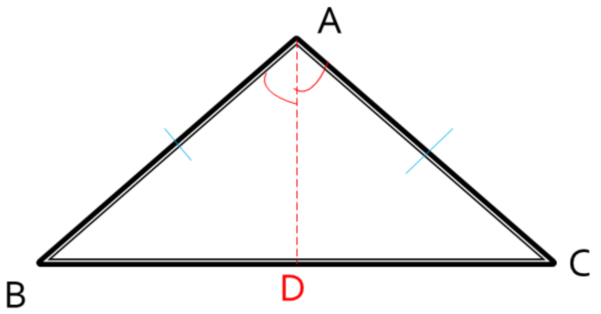


<u>Plan for Proof</u>:

- $< B$ and $< C$ are __$\cong$__ by using ___$CPCTC$___.
- To get the two triangles we need, we have to bisect $< A$ with $\overline{AD}$.
- Thus, the Diagram suggests that you first prove _ΔBAD_ $\cong$ _ΔCAD_.

Example : Complete this Proof

With this setup, we can prove this theorem in the following way:



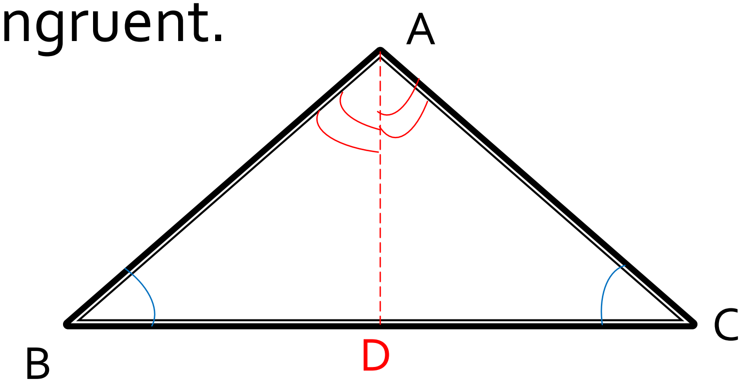
Statements	**Reasons**
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\overline{AD}$ bisects $< A$	2. Each angle has unique bisector
3. $<\mathrm{BAD} \cong < CAD$	3. Def. Angle Bisector
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\Delta BAD \cong \Delta CAD$	5. SAS Postulate
6. $< B \cong < C$	6. CPCTC

Theorem 4-1: The Corollaries

- Theorem 4-1 produces 3 Corollaries:
  - Corollary 1: An equilateral triangle is also equiangular.
  - Corollary 2: An equilateral triangle has three 60° angles.
  - Corollary 3: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

Theorem 4-2

- **Theorem 4-2**: If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



Given: $< B \cong < C$

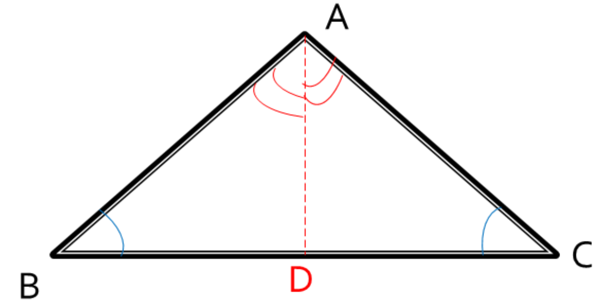
Prove: $\overline{AB} \cong \overline{AC}$

Plan for Proof:

- $\overline{AB}$ and $\overline{AC}$ are $\underline{\ \cong\ }$ by using $\underline{\quad CPCTC \quad}$.
- To get the two triangles we need, we have to bisect $< A$ with $\overline{AD}$.
- Thus, the Diagram suggests that you first prove $\underline{\Delta BAD}$ $\cong$ $\underline{\Delta CAD}$.

Example : Complete this Proof

With this setup, we can prove this theorem in the following way:



Statements	**Reasons**
1. $< B \cong < C$	1. Given
2. $\overline{AD}$ bisects $< A$	2. Each angle has a unique bisector
3. $< \mathrm{BAD} \cong < CAD$	3. Def. of Perp. Lines
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\Delta BAD \cong \Delta CAD$	5. AAS Theorem
6. $\overline{AB} \cong \overline{AC}$	6. CPCTC

Theorem 4-2: The Corollaries

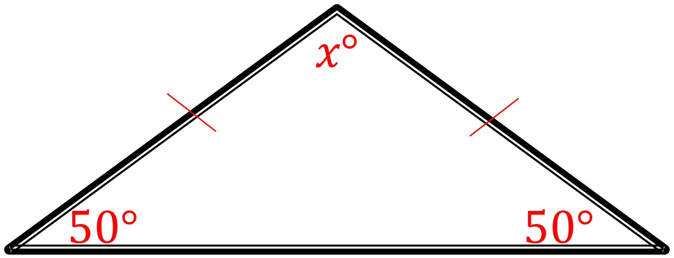
- Theorem 4-2 produces 1 Corollary:
  - Corollary: An equiangular triangle is also equilateral.

- But enough about proofs…

- Now that we know some properties of Isosceles triangles, we can use this knowledge to solve for variable in them.

Examples with Isosceles Triangles

- Solve for the value of x.



Using Theorem 4-1, we can say that the third angle in this triangle has measure __50°__.
With that, we can make an equation and solve:

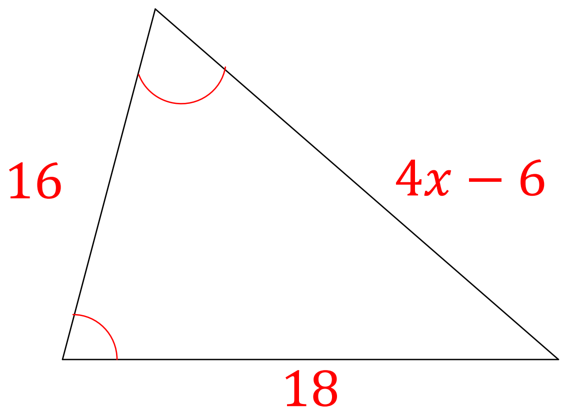
$$x + 50 + 50 = 180$$

$$x + 100 = 180$$

$$x = 80$$

Examples with Isosceles Triangles

- Solve for the value of x.



Using Theorem 4-2, we can set the sides opposite our congruent angles equal to each other, making the following equation:

$$18 = 4x - 6$$

$$24 = 4x$$

$$x = 6$$